NOISE AND DISTURBANCES

Ch.6

Design of low noise electronic circuits

In the design we can approach the noise in the following ways.
1) We favor the signal aspect optimizing the gain, the band, the stability, etc., and at the end we make an evaluation of the noise of the solution obtained. If the noise is too high, almost nothing can be done to reduce it, without a total redesign.
2) From the beginning, all the choices are oriented for minimizing the noise of the first stage , even if from the point of view of the signal, the solution is not the best. For example, if the impedance of the transducer is purely resistive, we will have to design a pre-amplifier having an optimal noise resistance equal to the resistance of the transducer for at least one frequency. Further, the noise is integrated into the required band, taking into account the graphs of variation of the noise produced by the transducer and those of the first floor according to the frequency. If the result is not satisfactory, the polarization point of the transistor or even the transistor must be changed.

The next step is to establish the polarization points and to calculate the following stages, as well as the coupling networks between stages. Finally, the total noise is re-evaluated to make sure it does not exceed the required goal. Subsequently, the negative reaction will be used to adjust the gain, band and impedance of input or output.
This second method is by far the most recommended.

Noise reduction in low frequency design

$$v_1 = 1mV\,, v_2 = 2mV$$

$$\sqrt{1^2 + 2^2} = \sqrt{5} = 2.23mV$$

$$\sqrt{1^2 + 0.3^2} = 1.044mV$$

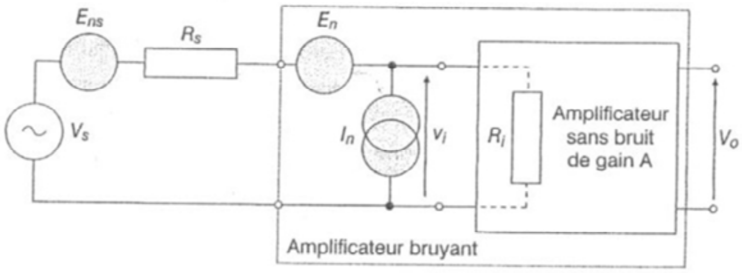


Fig.6.1

$$\overline{\left(v_1 + v_2\right)^2} = \overline{v_1^2} + 2\overline{v_1 v_2} + \overline{v_2^2} = \overline{v_1^2} + 2p\left(\overline{v_1^2}\right)\left(\overline{v_2^2}\right) + \overline{v_2^2} \quad (6.1)$$

We consider the typical case of a translator whose signal is processed by a pre-amplifier, possibly followed by a shaping circuit and other amplifying stages.

Design principles

1) If several sources of noise occur simultaneously in the same place, the designer must identify the dominant source and direct all design effort towards reducing it. Suppose that, in a given circuit, two noise sources v1 = 1 mV and v2 = 2 mV are present. The total value of the resulting noise will be Eq. (6.1) where we used p as the correlation coefficient (0 <p <1). Assuming the correlation is null, the total noise is √5 = 2.23 mV, which hardly represents an increase of 10% in relation to the dominant source. It is obvious that directing the effort to reduce source v1 is a wrong strategy.

2) The pre-amplifier noise is negligible to that of the translator if it represents at most 1/3 of the translator noise.
As these two sources are decorated, the relation (6.1) shows that in this case the total noise would be √ (1 ^ 2 + 〚0.3〛 ^ 2) ≅1.044, which justifies the statement. From a practical point of view, it is clear that reducing the noise of the pre-amplifier below this limit would not bring any sensible improvement.

3) In order to reduce the noise of a system, its frequency band must be limited to the minimum necessary to transmit useful signals.
At present, we are confronted with the continuous improvement of the frequencies performances of the electronic components, accompanied by the decrease of their price. For this reason, it is not difficult to obtain a bandwidth higher than that imposed by the specifications, almost without higher expenses.
However, we must remember that the thermal noise and the noise of alice have the power proportional to the band. An effective means of combating them is therefore to limit the band (for example by using a filter placed at the entrance of the circuit as much as possible). Particular attention should be paid to the 50 Hz frequency filtering and its harmonics, if they fall into the transmitted band

4) The most effective strategy to reduce the noise of a system is to reduce the sources located at the input (if possible, the noise of the signal source must be combated).
The signal source is either a transducer (pressure, humidity, temperature, etc.) or an antenna (in case of space communications, radar or telecommunications systems).
As they occupy the first position in relation to the propagation direction of the signal, their noise is amplified by all the next floors. Thus, choosing a less noisy translator (or a more directional and better tuned antenna) is the most effective solution to reduce noise.

5) To reduce the noise of a system, its temperature must be reduced significantly.
This solution is the most expensive. It is used at the last moment, when all other means used to reduce the noise have been exhausted. Then there remains the cryogenic solution, that is, maintaining the circuit in a thermostat at a temperature of several tens of degrees Kelvin. This technique is used in systems that receive very weak signals (radio astronomy, space communications, etc.).

Characterization of amplifiers

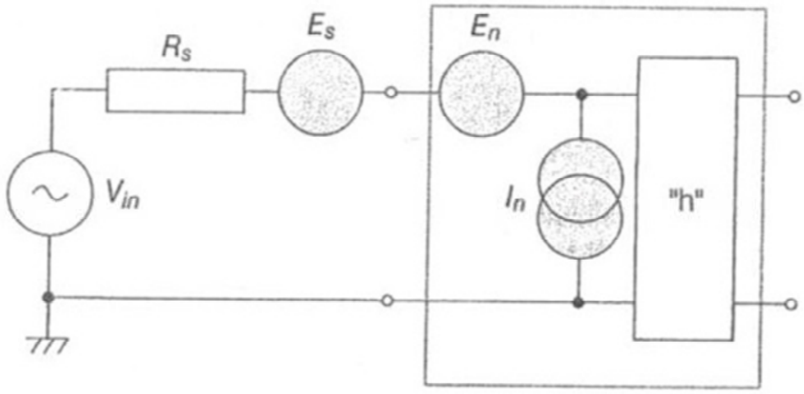


Fig.6.2

$$S\left(E_{ni}\right) = 4kTR_s + \overline{E_n^2} + \overline{I_n^2}R_S^2 \quad (6.2)$$

The spectral density of the total noise voltage at the input of the amplifier is Eq. (6.2). This value is important in design because it decides the minimum signal level we can amplify.

Characterization of amplifiers

$$\left(\frac{S}{N}\right)_i = 20\log\left(\frac{V_s}{E_{ni}}\right) = 10\log\frac{V_s^2}{\left(4kTR_s + \overline{E_n^2} + \overline{I_n^2}R_S^2\right)\Delta f} \quad (6.3)$$

$$F = 1 + \frac{\overline{E_n^2} + \overline{I_n^2}R_S^2}{4kTR_s} \quad (6.4a)$$

$$F(dB) = 10\log\left(1 + \frac{\overline{E_n^2} + \overline{I_n^2}R_S^2}{4kTR_s}\right) \quad (6.4b)$$

The signal-to-noise ratio at the input, expressed in dB, is given by the relation (6.3).
The noise factor will be given by the relation (6.4a), and in dB by the relation (6.4b).

Characterization of amplifiers

$$F_o = 1 + \frac{E_n I_n}{2kT} \quad (6.5a) \qquad R_s = R_o = \frac{E_n}{I_n} \quad (6.5b)$$

$$F = 1 + \frac{F_o - 1}{2}\left(\frac{R_s}{R_o} + \frac{R_o}{R_s}\right) \quad (6.6)$$

Deriving the expression (6.4a) in respect to Rs, we find the minimum noise factor, Eq. (6.5a). It is reached when the optimal resistance of the source is given by the relation (6.5b).
Using the relations (6.4a) and (6.5), we deduce the final expression of the noise factor, (6.6).

Case study 1

We have a transducer that provides a signal of 0.18 µV (effective value) in a band 10 Hz around the frequency of 3 kHz. Its internal resistance is Rs = 1 kΩ. To amplify the signals from this transducer, we intend to design a pre-amplifier using transistor 2N930, in EC connection. With the help of measurements, we found that at a colector current of 1 mA and at a frequency of 3 kHz, the equivalent noise generators are:

En = 3 nV / $\sqrt{Hz}$ and In = 5 pA / $\sqrt{Hz}$

Which is the best signal-to-noise ratio we can get at the input, using this transistor, polarized to 1 mA?

Answer

$$\overline{E_{ni}^2} = \left(4kTR_s + \overline{E_n^2} + \overline{I_n^2}R_S^2\right)\Delta f =$$

$$= \left(16\times10^{-21}\left(10^3\right) + \left(3\times10^{-9}\right)^2 + \left(10^3\times5\times10^{-12}\right)^2\right)10 = 5\times10^{-16}V^2 \quad (6.7)$$

$$\frac{S_i}{N_i} = 20\log\left(\frac{V_{in}}{E_{ni}}\right) = 20\log\left(\frac{180nV}{22.3nV}\right) \cong 18.12\ dB \quad (6.8)$$

Case study 2

The noise of a field effect transistor is described in the catalog by :
En = 0.03 µV / √Hz and In = 0.15 pA / √Hz, at a frequency of 1 kHz, the generators being supposedly uncorrelated.
Which is the optimum resistance of the source and the minimum noise factor who fits them?
2) If the signal source is a transducer having internal resistance of 100 kΩ, which is the minimum noise factor of a preamplifier who uses this transistor?

Answer

$$R_o = \frac{E_n}{I_n} = \frac{0.03\times10^{-6}}{0.15\times10^{-12}} = 200\ k\Omega \quad (6.9)$$

$$F_o = 1+\frac{E_n I_n}{2kT} = \frac{\left(0.03\times10^{-6}\right)\left(0.15\times10^{-12}\right)}{2\left(1.38\times10^{-23}\left(J/K\right)\right)\left(290\right)} = 1.56 = 1.94\ dB \quad (6.10)$$

$$F = 1+\frac{F_o-1}{2}\left(\frac{R_s}{R_o}+\frac{R_o}{R_s}\right) = 1+\frac{0.56}{2}\left(\frac{100}{200}+\frac{200}{100}\right) = 1.7 = 2.3\ dB \quad (6.11)$$

1) The optimal resistance is calculated with the relation (6.9). Suppose here that this transistor equips the first stage of the pre-amplifier.
The minimum noise factor is given by the relation (6.10).

2) For a 100 kΩ source resistance, we use the relation (6.11).

As this value is sufficiently different from the optimal noise factor, we can think of the wrong transistor being used. A change in the value of its current or polarization would result in the change of the values of the equivalent noise generators at the input and, consequently, a proximity between Ro and the transducer resistance (which is usually fixed in advance). If not, the transistor must be changed.

Transducer-network coupling-amplifier

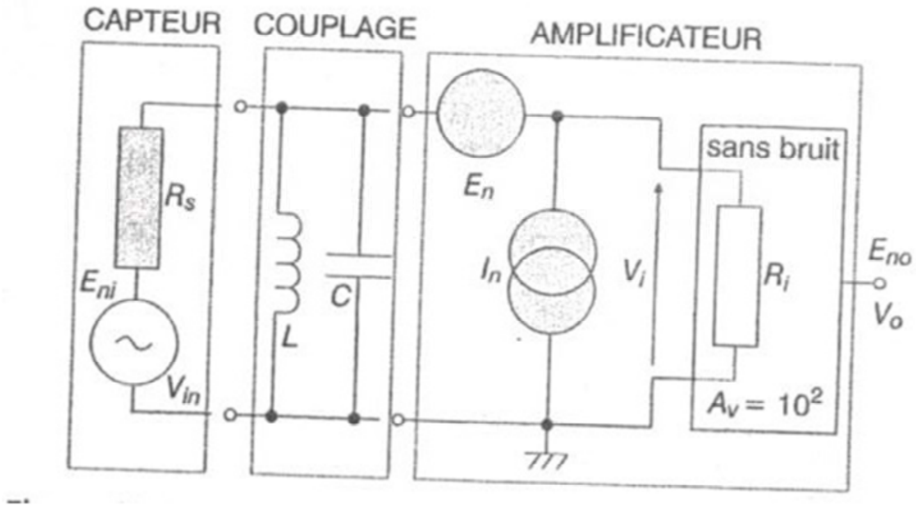


Fig.6.3

We consider the system of Fig. 6.3, where L = 1 mH, C = 1 µF. The transducer has an internal resistance of Rs = 1 kΩ and provides an effective value signal Vin = 10 µV. Suppose that T = 300 K. The noise of the amplifier is characterized , at the frequency of 10 kHz , by the sources: En = 20 nV / $\sqrt{Hz}$, In = 3 nA / $\sqrt{Hz}$ (supposedly uncorrelated) and by an input resistance Ri = 10 kΩ.

Calculate the equivalent noise at the input and output (Eni and Eno), in a 1 Hz band, around the 10 kHz frequency.

Transducer-network coupling-amplifier
Small signal analysis

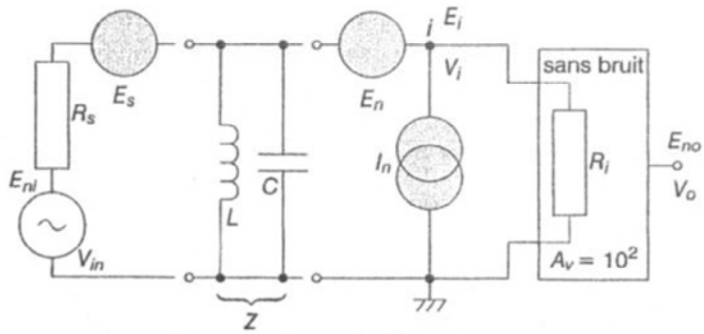


Fig.6.4

$$V_o = A_v V_i \quad (6.12a)$$

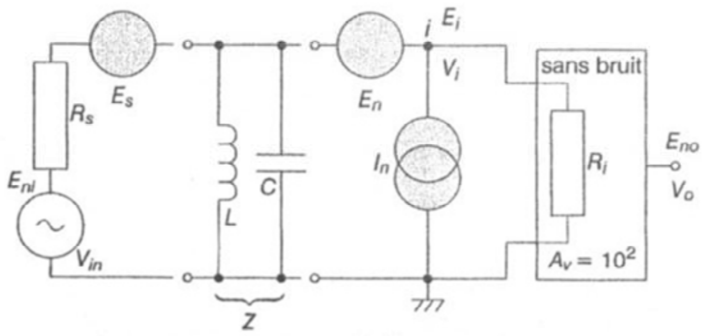
$$V_o = A_v \underbrace{V_{in} \frac{R_i \parallel Z}{R_s + R_i \parallel Z}}_{V_i} \quad (6.12b)$$

$$K = \frac{V_o}{V_{in}} = A_v \frac{R_i \parallel Z}{R_s + R_i \parallel Z} \quad (6.12c)$$

All signal voltages are noted (Vin, Visi Vo). The noise ones are noted: Es (thermal noise associated with the source resistance), En (equivalent noise generator of the amplifier) and Ei, noise voltage in point i. We also note with Eno, Eni the equivalent noise voltages at the output, respectively at the input.
The equivalent scheme is given in Fig. 6.4.

We have Eq. (6.12a) where Vi and Vo are given by (6.12b). Therefore, the amplification of the circuit, from the point of view of the signal, between the generator at the input and the output of the amplifier, is (6.12c).

Transducer-network coupling-amplifier Noise analysis



$$E_{no} = A_v E_i \Rightarrow \overline{E_{no}^2} = |A_v|^2 \, \overline{E_i^2} \quad (6.13a)$$

Fig.6.4

$$E_i = I_n (R_i \parallel Z \parallel R_s) + E_n \frac{R_i}{R_i + Z \parallel R_s} + E_s \frac{R_i \parallel Z}{R_s + R_i \parallel Z} \quad (6.13b)$$

The sources of noise present in the circuit are: Es and the equivalent generators En and In (the input resistance is therefore considered noiseless). All these sources being uncorrelated, their spectral densities gather at the output.
From the equivalent scheme, we obtain (6.13a), because considering the average value of a complex quantity comes back to considering its module.
The noise voltage in point “I” is obtained by overlapping the contributions of all sources, in the 1 Hz band, Eq. (6.13b).

Transducer-network coupling-amplifier Noise analysis

$$S(E_i) = \overline{i_n^2}\left|R_i \parallel Z \parallel R_s\right|^2 + \overline{E_n^2}\left|\frac{R_i}{R_i + Z \parallel R_s}\right|^2 + \overline{E_s^2}\left|\frac{R_i \parallel Z}{R_s + R_i \parallel Z}\right|^2 \quad (6.13c)$$

$$S(E_{no}) = \left|A_v\right|^2\left(\overline{i_n^2}\left|R_i \parallel Z \parallel R_s\right|^2 + \overline{E_n^2}\left|\frac{R_i}{R_i + Z \parallel R_s}\right|^2 + \overline{E_s^2}\left|\frac{R_i \parallel Z}{R_s + R_i \parallel Z}\right|^2\right) \quad (6.13d)$$

$$S(E_{ni}) = \overline{i_n^2}\left|\frac{R_s + R_i \parallel Z}{R_i \parallel Z}\right|^2\left|R_i \parallel Z \parallel R_s\right|^2 + \overline{E_n^2}\left|\frac{R_i}{R_i + Z \parallel R_s}\right|^2\left|\frac{R_s + R_i \parallel Z}{R_i \parallel Z}\right|^2 + \overline{E_s^2} \quad (6.13e)$$

Therefore, the spectral density at point I will be Eq. (6.13c).
The output spectral density will be (6.13d).
In order to find the equivalent generator at the input, it is necessary to transpose this noise at the input by dividing the spectral density by | K | ^ 2, where K is the gain of the amplifier. Finally, we obtain the spectral density at the input (6.13e), where $E_s^2 = 4kTR_s$.

Transducer-network coupling-amplifier Numerical application

$$Z = \frac{(j\omega L)(1/j\omega C)}{(j\omega L)+(1/j\omega C)} \cong -j21.3$$

$$R_i \,||\, Z \simeq Z \simeq -j21.3 \quad |R_i||Z|^2 \simeq 454$$

$$R_s + R_i \,||\, Z \simeq 1000 - j21.3 \quad |R_s + R_i||Z|^2 \simeq 10^6$$

$$|R_i||Z||R_s|^2 \simeq 454 \quad |R_s + R_i|^2 |Z| \simeq 10^8 \,(6.14)$$

$$S(E_{ni}) \simeq 9\times10^{-12} + 0.88\times10^{-12} + 16.54\times10^{-18} \simeq 9.88\times10^{-12}\, V^2/Hz \quad (6.15)$$

$$E_{ni} \simeq 3.14\mu V/\sqrt{Hz} \quad (6.16) \qquad K = 100\frac{-j21.3}{1000-j21.3} \Rightarrow |K|^2 \simeq 4.54 \Rightarrow |K| = 2.13 \quad (6.17)$$

$$E_{no} = |K|E_{ni} \simeq 6.69\mu V/\sqrt{Hz} \quad (6.18)$$

$$S_i/N_i = V_{in}/E_{ni} = 20\log(10/3.14) \simeq 10\ dB \quad (6.19)$$

To make the numerical calculations, first calculate the quantities (6.14). In this case, the input noise density will be (6.15)

It is found that the thermal noise of the source resistance is negligible compared to that due to the generator In, which is of the same order of magnitude, but lower than En.

Starting from the value of the spectral density, we deduce (6.16).

For the signal, the overall gain at 10 kHz is (6.17). So Eno will be given by (6.18), and the signal-to-noise ratio by the relation (6.19).

The Attenuator

$$F = L = \frac{1}{G_a} \quad (6.20)$$

$$T_e = (L-1)T = \left(\frac{1}{G_a} - 1\right)T \quad (6.20a) \qquad T_e = (F-1)T_0 \quad (6.20b)$$

$$F = 1 + \frac{T}{T_0}\left(\frac{1}{G_a} - 1\right) \quad (6.20c)$$

$$F = \frac{1}{G_a} \quad (6.20d)$$

Theorem

The noise factor of a passive quadripole is equal to the attenuation introduced by this quadripole, Eq. (6.20), where Ga is the available power gain and L is the attenuation.

Demonstration

From relation (4.13), the equivalent temperature at the input of the quadripole is (6.20a), where T represents the quadripole temperature.
On the other hand, the expression (3.58) establishes the passage between the noise temperature and the noise factor, Eq. (6.20b).
Equating the expression (6.20a) with (6.20b), we obtain (6.20c)
If the quadripole is at the reference temperature, it results (6.20d).

Conclusion

The noise factor of any passive quadripole is equal to its attenuation, if and only if the quadripole as well as the generator dipole connected to its input are at reference temperatures.

Case study 3

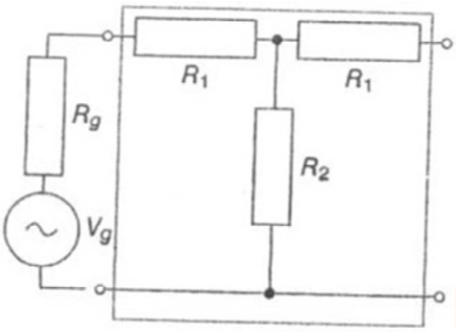


Fig.6.5

$$P_1 = V_g^2 / 4R_g \quad (6.21a)$$

$$V_e = V_g \frac{R_2}{R_1 + R_2 + R_g}, R_e = R_1 + \frac{(R_1 + R_g)R_2}{R_1 + R_g + R_2} \quad (6.21b)$$

$$F = \frac{3(50)(150(50) + 50(100))}{50^2 50} = 15 = 11.76\ dB \quad (6.21e)$$

$$P_2 = V_g^2 \frac{R_2^2}{(R_1 + R_2 + R_g)} \frac{1}{4((R_1 + R_g + R_2)R_1 + R_2(R_1 + R_g))} \quad (6.21c)$$

$$F = \frac{1}{G_a} = \frac{P_1}{P_2} = \frac{(R_1 + R_2 + R_g)((R_1 + R_g + R_2)R_1 + R_2(R_1 + R_g))}{R_g R_2^2} \quad (6.21d)$$

Consideram atenuatorul rezistiv din Fig.6.5., caracterizat de R1 = R2 = Rg = 50 Ω. Calculati factorul sau de zgomot.
We consider the resistive attenuator of Fig.6.5., decribed by R1 = R2 = Rg = 50 Ω. Calculate its noise factor.

Solution

We have to calculate the available power gain of the attenuator, defined as the ratio between the available power at the output (P2) and the power available from the generator (P1).
The power available from the generator, P1, is given by the relation (6.21a).
In order to calculate the available power P2 at the output of the quadripole, you must first calculate the Thevenin equivalent generator at the output. This generator is given in Eq. (6.21b).
The power available at the output, P2, will be (6.21c).
Thus, we reach the relationship (6.21d).
Substituting with the proposed numerical values, we obtain (6.21e).

Case study 4

Amplifier	Gain Power	Noise figure
A	6 dB	1.7
B	12 dB	2.0
C	20 dB	4.0

$$G_A = 4, G_B = 15.85, G_C = 100 \quad (6.22a)$$

$$F = F_A + \frac{F_B - 1}{G_A} + \frac{F_C - 1}{G_A G_B} = 1.7 + \frac{2-1}{4} + \frac{4-1}{4 \times 15.85} \simeq 1.997 \quad (6.22b)$$

$$F = F_A + \frac{F_C - 1}{G_A} + \frac{F_B - 1}{G_A G_C} = 1.7 + \frac{4-1}{4} + \frac{2-1}{4 \times 100} \simeq 2.45 \quad (6.22c)$$

Three different amplifiers, all adapted to input and output, have the characteristics given in the table.
These amplifiers are connected in cascade.
Indicate in what order we have to place these amplifiers to obtain the lowest noise and determine the minimum noise factor in this case.

Solution

According to the principles set out in this course, the first stage must be chosen with the lowest noise; in our case it is amplifier A. Then we will have the following possibilities:
1) A, B, C
2) A, C, B.

Before proceeding to the calculation we must express the gains in the form of a report, Eq. (6.22a)
Order A, B, C: Eq. (6.22b)
Order A, C, B: Eq. (6.22c)

In conclusion, the optimal order is A, B, C; this allows to obtain an overall noise factor of 1.997.

Case study 5

An amplifier has a voltage gain equal to 3 and an input resistance Ri = 5 kΩ. Its load is RL = 10 kΩ. Its first stage has a transistor described by an equivalent noise resistance Rn = 1.5 kΩ. What is the noise equivalent resistance defined at the input of the amplifier?

$$\overline{E_{ni}^2} = 4kT_0\Delta f\left(R_i + R_L/A_v^2 + R_n\right) \quad (6.23a)$$

$$\overline{E_{ni}^2} = 4kT_0\Delta f R_{eq} \quad (6.23b)$$

$$R_{eq} = R_i + R_L/A_v^2 + R_n = 5 + 10/9 + 1.5 \simeq 7.61\, k\Omega \quad (6.23c)$$

Solution

According to the definition, the equivalent noise resistance at the input is the physical resistance that connected to the input of an identical, but ideal (noiseless) amplifier, would produce at the output the same noise power as the real system.

In the case of our problem, we have three sources of noise: input resistance, the load and noise of the first stage (modeled by Rn). All these sources are uncorrelated and their contributions must be transposed at the input. Thus, the mean square value of the equivalent noise voltage at the input will be Eq. (6.23a), where Av is the voltage gain.

Equivalent noise resistance must produce the same mean square value, at temperature T0, so we have Eq. (6.23b). From this, we deduce Eq. (6.23c)

Case study 6

A multi-stage amplifier has the following noise parameters:
Rn = 20 Ω, Gn = 6.4 mS if Ycor = (2 + j14) mS. The signal source has an internal resistance of 50 Ω and delivers a signal at 8 GHz.
What is the noise factor of the amplifier?

$$F = 1 + \frac{G_n + R_n\left(\left(G_s + G_{cor}\right)^2 + \left(B_s + B_{cor}\right)^2\right)}{G_s} =$$

$$= 1 + \frac{6.4 + 0.02\left(\left(20+2\right)^2 + \left(0+14\right)^2\right)}{20} = 2.0 \quad (6.24)$$

Using equation (4.22a), repeated on the slide, the noise factor will be given by the relation (6.24).

Case study 7

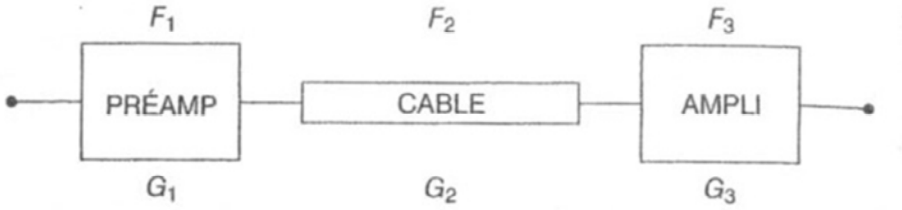


Fig.6.6

Preamplificator	Cablu	Amplificator
6 dB	8 dB	13 dB

$$F_1 = 6\ dB = 3.98$$
$$F_2 = 8\ dB = 6.3 \qquad (6.25a)$$
$$F_3 = 13\ dB = 19.95$$
$$F = 9\ dB = 7.94$$
$$G_2 = 1/L_2 = 1/6.3 \simeq 0.158$$

$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} \qquad (6.25b)$$

$$G_1 = \frac{1}{F - F_1}\left((F_2 - 1) + \frac{F_3 - 1}{G_2}\right) = \frac{1}{3.98}\left((6.3 - 1) + \frac{19.95 - 1}{1/6.3}\right) \simeq 31.5 \qquad (6.25c)$$

$$G_1 \geq 15\ dB$$

A receiver has an antenna preamplifier connected by a long cable to the amplifier. The noise factors of these components are given in the table. Calculate the minimum gain of the preamplifier, knowing that the overall noise factor of the receiver should not exceed 9 dB.

The solution

In a first step, we must transform all the values given in dB as reports, Eq. (6.25a). We appeal to the relation (3.59), repeated for this case in Eq. (6.25b). Finally we have the relationship (6.25c)

Case study 8

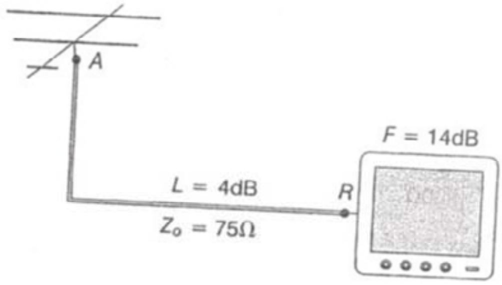


Fig.6.7

$$F = 10\log\left(\overline{E_{nR}^2}/\overline{E_g^2}\right)[dB] \quad (6.26a)$$

$$\overline{E_{nR}^2} = 4kTZ_0\Delta f 10^{F/10} = 4\left(1.38\times10^{-23}\right)(290)(75)\left(4\times10^6\right)\times10^{1.4} \simeq 1.206\times10^{-10}V^2 \quad (6.26b)$$

$$S_R^2 = 10^4\,\overline{E_{nR}^2} = 1.206\times10^{-6}V^2 \Rightarrow S_R = 1.098\,mV \quad (6.26c)$$

$$E_A = S_R\cdot L = (1.098)(2.511)\,mV \simeq 2.758\,mV \quad (6.26c)$$

A TV is connected to the antenna by a coaxial cable (Fig. 6.7) with a characteristic impedance of 75 Ω, which introduces L = 4 dB attenuation. The TV is characterized by a noise factor of 14 dB and an input band of 4 MHz.
What has to be the minimum signal detected at the antenna terminals, if a good functioning of the TV requires a signal / noise ratio of at least 40 dB at its input?

Solution

If we apply the definition of North to the noise factor, all the powers being transposed at the input of the receiver (point R), we can write the relation (6.26a), where (E_nR ^ 2)‾ represents the mean square value of the total noise voltage at the input of the TVset (the source noise included) and (E_g ^ 2)‾ is the source contribution (in this case, 4kTZoΔf, assuming that in the working band, the imaginary part of the characteristic impedance of the cable is null). Follow Eq. (6.26b).
Since the signal / noise ratio at point R is imposed (40 dB or 10 ^ 4), by its definition, we can calculate the required signal value, Eq. (6.26c).
The minimum signal we need in point A is the signal obtained in point R, multiplied by the attenuation of the cable, Eq. (6.26d).