NOISE AND DISTURBANCES

Ch.5

Calculation of noise parameters for a linear circuit

The calculation method must satisfy the following constraints:
1) The topology of the circuit to be analyzed must be completely arbitrary
2) The noise of each active device (transistor) is described by the sizes (Fo, Rn, Go and Bo) or any other equivalent assembly; the electrical behavior is characterized by the parameters Y say S.
3) Each passive device is assumed to generate only thermal noise.

As a result of the analysis, we will have to obtain the noise parameters of the global circuit. If the circuit has a single input and a single output it will be assimilated to a quadripole, characterized by the four noise parameters or by the correlation matrix between the two equivalent sources of noise at the input.

If the circuit has more inputs or outputs, then the only means of characterization remains the correlation matrix between the equivalent sources of noise at the ports.

Calculation of noise parameters for a linear circuit

- **Hillbrand and Russer Methode**

$$\mathbf{C}_Z = \mathbf{C}_{Z1} + \mathbf{C}_{Z2} \quad (5.1)$$

$$\mathbf{C}_Y = \mathbf{C}_{Y1} + \mathbf{C}_{Y2} \quad (5.2)$$

$$\mathbf{C}_A = \mathbf{A}_1 \mathbf{C}_{A2} \mathbf{A}_1^+ + \mathbf{C}_{A1} \quad (5.3)$$

$$\mathbf{C}_Z = 2kT\Re(\mathbf{Z}) \quad (5.4)$$

$$\mathbf{C}_Y = 2kT\Re(\mathbf{Y}) \quad (5.5)$$

$$\mathbf{C}_A = 2kT \begin{bmatrix} R_n & \frac{F_o - 1}{2} - R_n Y_o^* \\ \frac{F_o - 1}{2} - R_n Y_o & R_n |Y_o|^2 \end{bmatrix} \quad (5.6)$$

Hillbrand and Russer developed an approach based on quadripole analysis. The simulated circuit is split into a number of elementary quadruples, whose electrical and noise performances are easy to evaluate.
Depending on the way in which the elementary quadripoles are interconnected (in series, in parallel or in the cascade), the most convenient description is adopted (using the parameters Z, Y or ABCD). For example, assuming we have two elementary quadruples, denoted by the indication "1" and "2", the global correlation matrix is obtained with the expressions (5.1) for the serial connection, (5.2) for the parallel connection and (5.3) for the cascade connection. . In Eq. (5.3) A is the chain matrix of the quadripole and A ^ + is attached to it.

We recall that for each quadripole, depending on the representation adopted, the correlation matrix is calculated with the relations (5.4) - (5.6).

Calculation of noise parameters for a linear circuit

- **Hillbrand and Russer Methode**

$$\mathbf{C}^{+} = \mathbf{T}\mathbf{C}\mathbf{T}^{+} \quad (5.7)$$

Table1

T	**Admittance**	**Impedance**	**Chane**
Admittance	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix}$	$\begin{bmatrix} -y_{11} & 1 \\ -y_{21} & 0 \end{bmatrix}$
Impedance	$\begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$	$\begin{bmatrix} 1 & -z_{12} \\ 0 & -z_{22} \end{bmatrix}$
Chane	$\begin{bmatrix} 0 & a_{12} \\ 1 & a_{22} \end{bmatrix}$	$\begin{bmatrix} 1 & -a_{11} \\ 0 & -a_{21} \end{bmatrix}$	$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$

To move from one representation to another, we must look for the correlation matrix, denoted C ^ +, with the relation (5.7), where C is the correlation matrix of the original representation, T is a transformation matrix, which is chosen according to the Table 1.

The noise parameters are obtained from the global matrix C_A, using the expressions (4.33).

The study of an attenuator

- **Configuration**

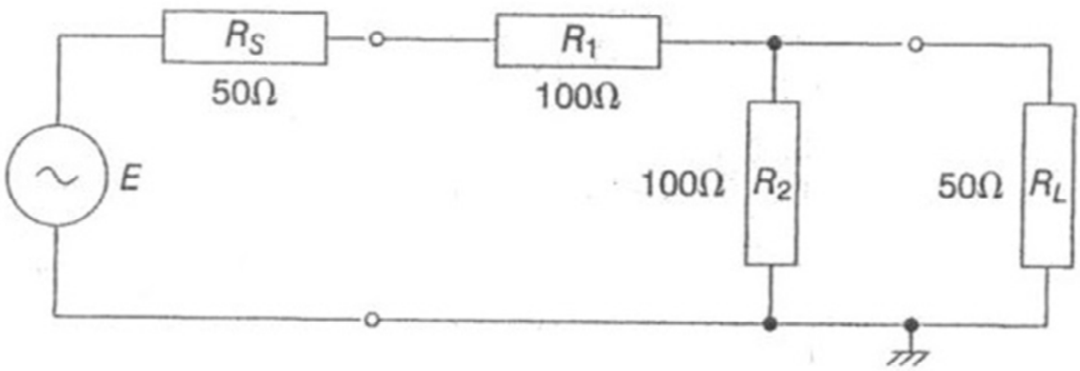


Fig.5.1

The proposed attenuator is shown in Fig.5.1.

The study of an attenuator

- **Q1 calculus**

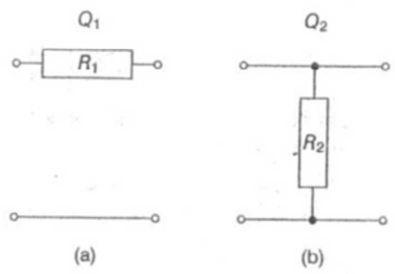


Fig.5.2

$$\mathbf{Y}_1 = \begin{bmatrix} 1/R_1 & -1/R_1 \\ -1/R_1 & 1/R_1 \end{bmatrix} \quad (5.8)$$

$$\mathbf{C}_{Y1} = 2kT\Re\{\mathbf{Y}_1\} = 2kT\begin{bmatrix} 1/R_1 & -1/R_1 \\ -1/R_1 & 1/R_1 \end{bmatrix} \quad (5.9a)$$

$$\mathbf{A}_1 = \begin{bmatrix} 1 & R_1 \\ 0 & 1 \end{bmatrix} \quad (5.9b) \quad \mathbf{T}_1 = \begin{bmatrix} 0 & a_{12} \\ 1 & a_{22} \end{bmatrix} \quad (5.9c)$$

$$\mathbf{C}_{A1} = 2kT\begin{bmatrix} 0 & R_1 \\ 1 & 1 \end{bmatrix}\begin{bmatrix} 1/R_1 & -1/R_1 \\ -1/R_1 & 1/R_1 \end{bmatrix}\begin{bmatrix} 0 & 1 \\ R_1 & 1 \end{bmatrix} = 2kT\begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix} (5.10)$$

The calculation of the noise parameters is performed by observing that the circuit consists of two elementary quadruples connected in the cascade, the diagram of which is illustrated in Fig.5.2.
We look for the global correlation matrix and after the relation (5.3) it is sufficient to calculate the chain matrices of two quadruples, as well as the corresponding correlation matrices. We have four stages:
The study of quadripole Q1
The quadripole shown in Figure 5.2a is easily characterized by its matrix Y1, Eq. (5.8).
With the help of the relation (5.5) we have Eq. (5.9a).
Using Table 1 we have the Relationship (5.9b).
The transition to the correlation matrix in the chain representation is performed according to the expression (5.7), adopting the corresponding transformation matrix from Table 1, ie Eq. (5.9c), which leads to Eq. (5.10).

The study of an attenuator

- **Q2 calculus**

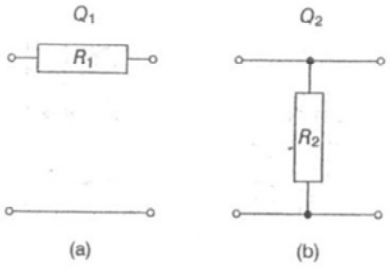


Fig.5.2

$$\mathbf{Z}_2 = \begin{bmatrix} R_2 & R_2 \\ R_2 & R_2 \end{bmatrix} \quad (5.11a)$$

$$\mathbf{A}_2 = \begin{bmatrix} 1 & 0 \\ 1/R_2 & 1 \end{bmatrix} \quad (5.11b)$$

$$\mathbf{C}_{Z2} = 2kT\Re\{\mathbf{Z}_2\} = 2kT\begin{bmatrix} R_2 & R_2 \\ R_2 & R_2 \end{bmatrix} \quad (5.11c)$$

$$\mathbf{T}_2 = \begin{bmatrix} 1 & -a_{11} \\ 0 & -a_{21} \end{bmatrix} = \begin{bmatrix} 1 & -1 \\ 0 & -1/R_2 \end{bmatrix} \quad (5.11d)$$

$$\mathbf{C}_{A2} = 2kT\begin{bmatrix} 1 & -1 \\ 0 & -1/R_2 \end{bmatrix}\begin{bmatrix} R_2 & R_2 \\ R_2 & R_2 \end{bmatrix}\begin{bmatrix} 1 & 0 \\ -1 & -1/R_2 \end{bmatrix} = 2kT\begin{bmatrix} 0 & 0 \\ 0 & 1/R_2 \end{bmatrix} (5.12)$$

For the quadripole defined in Figure 5.2b, the most convenient description is made using the matrix (5.11a), from which we have the matrix (5.11b) and (5.11c).
Then, the transition to the representation through the chain matrix is possible due to the transformation matrix T2 (see also Table 1), Eq. (5.11d), which leads us to the relation (5.12).

The study of an attenuator

- **Calculation - cascading**

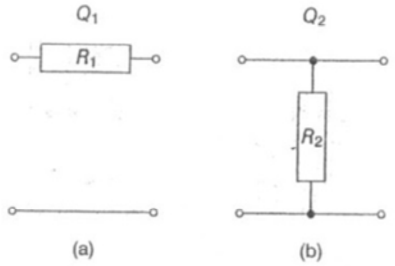


Fig.5.2

$$\mathbf{C}_A = 2kT\left\{\begin{bmatrix}1 & R_1\\0 & 1\end{bmatrix}\begin{bmatrix}0 & 0\\0 & 1/R_2\end{bmatrix}\begin{bmatrix}1 & 0\\R_1 & 1\end{bmatrix}+\begin{bmatrix}R_1 & 0\\0 & 0\end{bmatrix}\right\} = 2kT\begin{bmatrix}\left(R_1^2/R_2\right)+R_1 & R_1/R_2\\R_1/R_2 & 1/R_2\end{bmatrix} \quad (5.13)$$

The global correlation matrix is calculated with the relation (5.3), substituting the expressions (5.9b), (5.12) and (5.10). We have Eq. (5.13).

The study of an attenuator

- **Calculation of noise parameters**

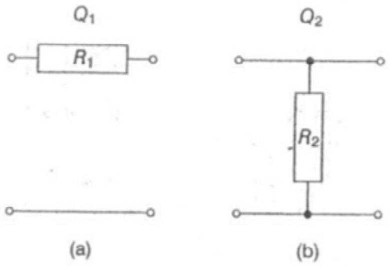


Fig.5.2

$$R_n = \left(R_1^2 / R_2\right) + R_1 \quad (5.14a)$$

$$Y_o = \sqrt{1/\left(R_1^2 + R_1 R_2\right)} \quad (5.11b)$$

$$F_o = 1 + 2\left(\frac{R_1}{R_2} + \left(\frac{R_1^2}{R_2} + R_1\right)\sqrt{\frac{1}{R_1^2 + R_1 R_2}}\right) \quad (5.14c)$$

$$F = 1 + \left(\frac{1}{R_S}\left(\frac{R_1^2}{R_2} + R_1\right) + \frac{R_1}{R_2} + \frac{R_1}{R_2} + \frac{R_S}{R_2}\right) \quad (5.14d)$$

The relations (4.33) give us the relations (5.14).

The study of an attenuator

- **Calculation of noise parameters**

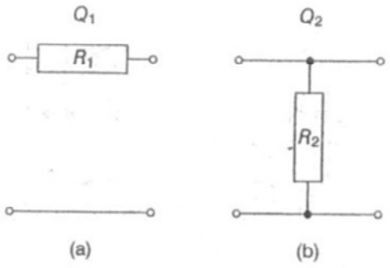


Fig.5.2

$$R_n = 2R \quad (5.15a)$$

$$Y_o = 1/R\sqrt{2} \quad (5.15b)$$

$$F_o = 1 + 2\left(1+\sqrt{2}\right) \quad (5.15c)$$

$$F = 1 + \left(\frac{2R}{R_S} + \frac{2R+R_S}{R}\right) \quad (5.15d)$$

$$R_n = 200\Omega, Y_o = (7.071 + j0)mS, F_o = 5.8284271(sau\ 7.6555dB), F = 7.5(sau\ 8.7506dB)$$

In the particular case R1 = R2 = R, the calculus relations become (5.15).
Using the numeric values we obtain the results on the slide.

- **Catalog of elementary circuits**

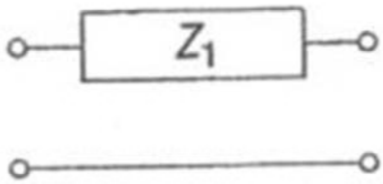


Fig.5.3a

$$\mathbf{C}_A = \begin{bmatrix} \Re(Z_1) & 0 \\ 0 & 0 \end{bmatrix}$$

R_n	$\Re(Z_1)$
Y_o	0
N	0
F_o	1

By applying the Hillbrand and Russer method to different configurations of simple quadripoles, the results obtained are on the slide.

- **Catalog of elementary circuits**

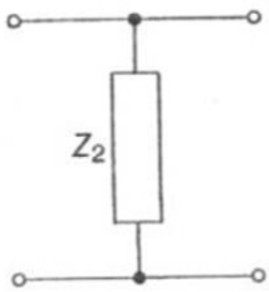


Fig.5.3b

$$\mathbf{C}_A = \begin{bmatrix} 0 & 0 \\ 0 & \dfrac{\Re(Z_2)}{Z_2 Z_2^*} \end{bmatrix}$$

R_n	0
Y_o	nedeterminat
N	0
F_o	1

By applying the Hillbrand and Russer method to different configurations of simple quadripoles, the results obtained are on the slide.

- **Catalog of elementary circuits**

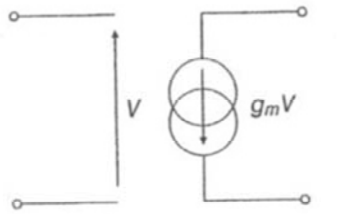


Fig.5.3c

R_n	0
Y_o	0
N	0
F_o	1

By applying the Hillbrand and Russer method to different configurations of simple quadripoles, the results obtained are on the slide.

- **Catalog of elementary circuits**

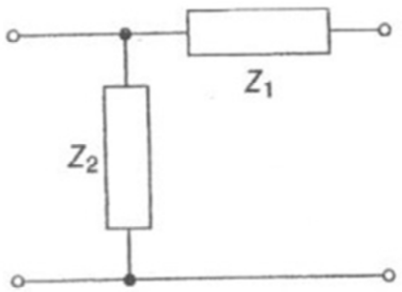


Fig.5.3d

$$\mathbf{C}_A = \begin{bmatrix} \Re(Z_1) & \Re(Z_1)/Z_2^* \\ \Re(Z_1)/Z_2 & \dfrac{1}{Z_2 Z_2^* \left(\Re(Z_1) + \Re(Z_2)\right)} \end{bmatrix}$$

By applying the Hillbrand and Russer method to different configurations of simple quadripoles, the results obtained are on the slide.

• **Catalog of elementary circuits**

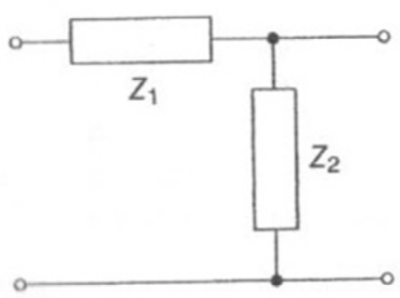


Fig.5.3e

$$\mathbf{C}_A = \begin{bmatrix} \frac{|Z_1|^2}{|Z_2|^2}\Re(Z_2)+\Re(Z_1) & \frac{Z_1}{Z_2 Z_2^*}\Re(Z_2) \\ \frac{Z_1^*}{Z_2 Z_2^*}\Re(Z_2) & \frac{1}{Z_2 Z_2^*}\Re(Z_2) \end{bmatrix}$$

By applying the Hillbrand and Russer method to different configurations of simple quadripoles, the results obtained are on the slide.

• The calculation method for a multipole

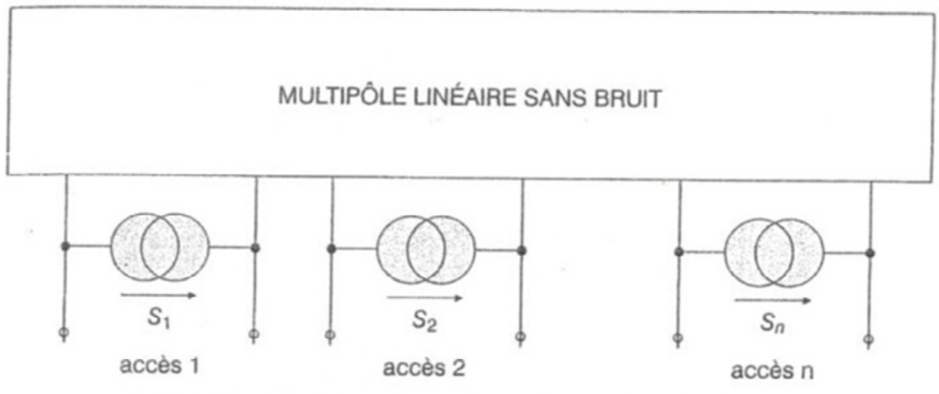


Fig.5.4

Modeling
The noise present at the terminals of a multipole comes from the thermal noise associated with each of its resistors, but also from the noise generated by each active device (transistor) that is inside. As usual, it is preferable to model outward noise through equivalent generators placed at the gates of an identical, supposedly ideal multipole.
In the present case, we prefer the Norton equivalent scheme (Figure 5.4), which, in a representation in terms of the Y parameters, is more suitable than the Thevenin representation.

Formulation
Let be a linear multipole having n external gates and containing m active devices. Each active device is characterized by its admittance matrix and the four noise parameters. We look for the correlation matrix of the Si sources (I = 1,2, ..., n) at the gates.

- **The calculation method for a multipole**

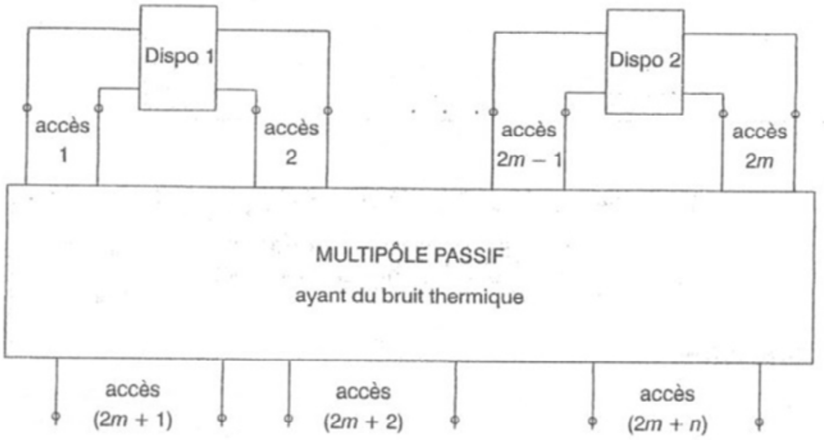
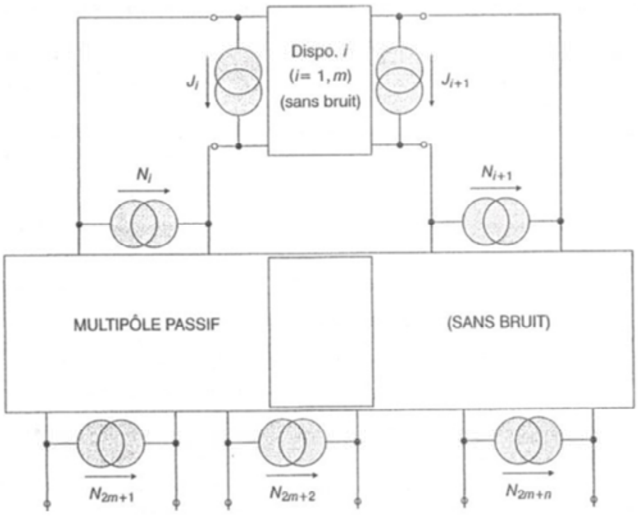


Fig.5.5

Principle
The approach chosen is to first extract all the active devices from the circuit, thus creating additional gates (according to figure 5.5).
Since the noise of the residual multipole is of a thermal nature and at the same time independent of the noise generated by the active devices, the total power of noise at the gates is calculated by overlapping these two contributions.

• The calculation method for a multipole



$$\mathbf{N}_N = \left[\left\langle N_p N_q^* \right\rangle \right] = 2kT\Delta f \left(\mathbf{Y} + \mathbf{Y}^+ \right) \quad (5.16)$$

Fig.5.6

Each noisy component in figure 5.5 is in turn replaced by a Norton representation (for quadripoles, according to figure 4.4c, and for dipoles according to figure 5.4).
We arrive at figure 5.6, in which two categories of equivalent noise generators appear:
1) The sources noted Nk (k = 1,2,…, 2m + n) that simulate the thermal noise of the passive multipole. They are all correlated, and their noise matrix is (5.16)
2) Ji sources (I = 1,2,…, 2m) represent the noise of active devices, which is not only of thermal nature. The sources Ji and J(i + 1) belong to the same device, they are correlated with each other, but the noise of the different pairs is not correlated (because each pair represents a distinct transistor).

- **The calculation method for a multipole**

$$\mathbf{Y} = \begin{bmatrix} \mathbf{Y}_{dd} & \mathbf{Y}_{de} \\ \mathbf{Y}_{ed} & \mathbf{Y}_{ee} \end{bmatrix} \quad (5.17)$$

$$\mathbf{y} = \begin{bmatrix} y^{(1)} & 0 & . & 0 \\ 0 & y^{(2)} & . & 0 \\ . & . & . & . \\ 0 & 0 & . & y^{(m)} \end{bmatrix} \quad (5.19)$$

$$\begin{cases} \mathbf{I}_d = \mathbf{Y}_{dd}\mathbf{V}_d + \mathbf{Y}_{de}\mathbf{V}_e + \mathbf{N}_d & (5.18a) \\ \mathbf{I}_e = \mathbf{Y}_{ed}\mathbf{V}_d + \mathbf{Y}_{ee}\mathbf{V}_e + \mathbf{N}_e & (5.18b) \\ \mathbf{I}_d = -\mathbf{y}\mathbf{V}_d - \mathbf{J} & (5.18c) \end{cases}$$

$$\begin{cases} -\mathbf{y}\mathbf{V}_d - \mathbf{J} = \mathbf{Y}_{dd}\mathbf{V}_d + \mathbf{N}_d & (5.20a) \\ \mathbf{S} = \mathbf{Y}_{ed}\mathbf{V}_d + \mathbf{N}_e & (5.20b) \end{cases}$$

$$\mathbf{S} = \mathbf{H}_N\mathbf{N} + \mathbf{H}_J\mathbf{J} \quad (5.21)$$

$$\mathbf{H}_J = -\mathbf{Y}_{ed}\left(\mathbf{Y}_{dd} + \mathbf{y}\right)^{-1} \quad (5.22)$$

$$\mathbf{H}_N = \left[\mathbf{H}_J \quad \mathbf{I}_n\right] \quad (5.23)$$

Procedure

To calculate the total noise power at the gates defined as outputs, we start by decomposing in blocks the matrix of the multipole Y, as in Eq. (5.17), where the index "d" denotes the 2m gates created by extracting the active devices, and the index "e" corresponds to the exterior gates.

Then, the equilibrium equations of the network are in (5.18), where:

[V] = $[[V_d] \quad [V_e]]^t$ is the vector of gate voltages

[I] = $[[I_d] \quad [I_e]]^t$ the vector of currents at gates

[N] = $[[N_d] \quad [N_e]]^t$ the vector of thermal noise sources

[J] is the vector of the noise sources associated with the active devices

The matrix [y] that appears in Eq. (5.18c) is the diagonal sum of all the individual admitting matrices $[y^{(i)}]$ having the active devices, that is, the relation (5.19).

The classic Norton generator calculation procedure from a multipole gate is to short-circuit the gate and to "measure" the current passing through it.

Thus, to evaluate the equivalent noise currents at the gates outside the multipole, we set the condition [V_e] = 0 and note [I_e] with [S].

Starting from the system (5.18) we can deduce (5.20) by eliminating Id.

Further, by eliminating Vd between the equations in the system (5.20), we obtain Eq. (5.21), where the first term of the sum represents the contribution of the passive network and the second, the contribution of all active devices.

One can prove the relation (5.22) and (5.23), where In is the ordinal identity matrix n. In relation (5.23), the matrix HJ is increased by the matrix In.

- **<u>The calculation method for a multipole</u>**

$$\left\langle \mathbf{SS}^* \right\rangle = \mathbf{H}_N \left\langle \mathbf{NN}^* \right\rangle \mathbf{H}_N^+ + \mathbf{H}_J \left\langle \mathbf{JJ}^* \right\rangle \mathbf{H}_J^+ \quad (5.24)$$

$$\left\langle \mathbf{SS}^* \right\rangle = 2kT\Delta f \left(\mathbf{H}_N \left(\mathbf{Y} + \mathbf{Y}^* \right) \mathbf{H}_N^+ + 2\mathbf{H}_J \mathbf{C}_J \mathbf{H}_J^+ \right) \quad (5.25)$$

$$\mathbf{C}_S = \mathbf{H}_N \left(\mathbf{Y} + \mathbf{Y}^* \right) \mathbf{H}_N^+ + 2\mathbf{H}_J \mathbf{C}_J \mathbf{H}_J^+ \quad (5.26)$$

$$\mathbf{Y}_L = \mathbf{Y}_{ee} + \mathbf{H}_J \mathbf{Y}_{de} \quad (5.27)$$

As mentioned earlier, the N and J generators are statistically independent and the noise matrix will result by superposition, Eq. (5.24).
Considering the expressions (5.16) and (4.32b) with index I, we have equation (5.25), where CJ represents the sum-diagonal matrix of all correlation matrices $[C\_J$ ^ $((i))]$ of the individual devices (it is constructed). in the same way as the matrix [y]).
The standardized correlation matrix is (5.26)

<u>Remarks</u>
1) Expression (5.26) offers the advantage of separating the noise contribution of the active devices from that of the residual passive multipol.
2) If the circuit has a single input and a single output (n = 2), after the calculation of the correlation matrix, one can proceed to the calculation of the global noise parameters., by means of the relations (4.35).
3) The global YL, multipole admittance matrix is calculated with relation (5.27) and gives us the possibility to estimate the voltage and power gains associated with the various gates, assuming that the impedances that are connected are known.
4) Moreover, from the YL matrix we can calculate the S matrix of the global circuit, with the classical conversion formulas between the Circuit Parameters.