NOISE AND DISTURBANCES

Ch.4
Circuit noise modeling

Noise that appears in an electronic circuit cannot be located in a specific place. To each resistor we must associate a source of noise and to each transistor we must associate 3 sources of noise.

In these conditions the analysis is very difficult and we usually perform a macro-modeling of the circuit, which will be regarded as an ideal dipole, quadripole or multipole, without any source of noise inside. At the same time, in order to ensure at its terminals the same voltage or current fluctuations as in the real circuit, we will have to add to its gates, in series or in parallel, equivalent noise generators. In turn, these generators can be represented by equivalent thermal noise sources and in this case we are talking about equivalent noise resistance, or an equivalent noise temperature.

In calculations, it is difficult to avoid the simultaneous presence of functions in the frequency domain (used to describe impedances) and functions in the time domain (associated with generators). Therefore, when making calculations at the mine, mixing between frequency-dependent and time-dependent quantities is inevitable.

Modeling the dipole noise

- **The case of a uniform temperature**

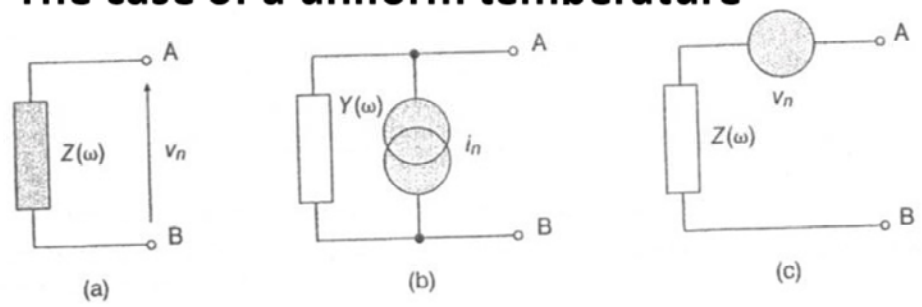


Fig.4.1

$$Z(\omega) = R(\omega) + jX(\omega) \quad (4.1)$$

$$Y(\omega) = 1/Z(\omega) = G(\omega) + jB(\omega) \quad (4.2)$$

$$\overline{v_n^2} = 4kTR\Delta f, R = \Re(Z) \quad (4.3a)$$

$$\overline{i_n^2} = 4kTG\Delta f, G = \Re(Y) \quad (4.3b)$$

We consider the dipole from (Fig.4.1a) which shows at its terminals the impedance (4.1), or the admittance (4.2).
Voltage fluctuations at terminals A and B are given by one or more noise sources found inside the dipole.
The equivalent circuit shown in (Fig.4.1b) is composed of a noise current generator parallel to the assumed dipole; by applying Thevenin's theorem, we can deduce its equivalent drawn in (Fig.4.1c).

Assuming that the original circuit has only thermal noise sources, all at the same temperature, the average quadratic value of the noise voltage, in a band Δf, around the frequency of measure f, is given by Nyquist's relation (4.3a), in while the equivalent noise current, its average quadratic value, is given by (4.3b).

Modeling the dipole noise

- **The case of a uniform temperature - Example**

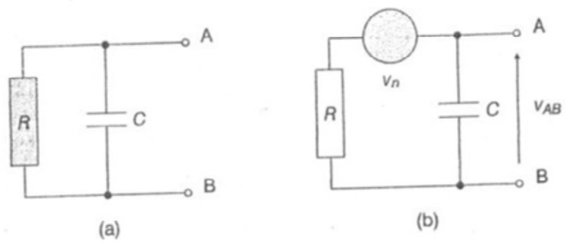


Fig.4.2

$$\overline{v_{AB}^2} = \frac{\overline{v_n^2}}{1+\omega^2 R^2 C^2} = \frac{4kTR\Delta f}{1+\omega^2 R^2 C^2} \quad (4.5)$$

$$v_{AB}^2 = \frac{v_n^2}{(1+j\omega RC)^2} \quad (4.4b)$$

$$v_{AB} = v_n \frac{1/j\omega C}{R+1/j\omega C} = \frac{v_n}{1+j\omega RC} \quad (4.4a)$$

Fie un circuit RC parallel ca in (Fig.4.2a). Interesul nostrum este pentru tensiunea de zgomot de la bornele A-B.
Aplicind formula divizorului de tensiune modelului Thevenin din (Fig.4.2b), avem relatia (4.4a).
Ridicind la patrat avem relatia (4.4b).
Pentru a trece la valori patratice medii ne amintim ca trebuie considerate valoarea medie a unei cantitati complexe, ceea ce revine la a considera modulul sau, Eq(4.5)

Modeling the dipole noise

- **The case of different temperatures**

Pierce's rule

$$T_a = a_1T_1 + a_2T_2 + a_3T_3 + \cdots \quad (4.6)$$

Pettai's rule

$$T_{eff} = a_1T_1 + a_2T_2 + a_3T_3 + \cdots \quad (4.7a)$$

$$a_1 + a_2 + a_3 + \cdots = 1 \quad (4.7b)$$

This situation is met, in practice, in two situations:
1) When we have different physical temperatures in different regions of the circuit (as in the case of space systems where the transducer is on the outside of the ship while the associated equipment is inside)
2) When describing a telecommunication system through a cascade of blocks characterized by different noise equivalent temperatures.

Pierce's rule
For an antenna used for emission, either a1 power fraction that is absorbed by a body at temperature T1, a2 power fraction that is absorbed by a body at temperature T2, a3 power fraction that is absorbed by a body at temperature T3, etc. Then, the temperature Te of the radiation resistance of the antenna is given by the expression (4.6).

Pettai's rule
Let a noise source that delivers unitary power to a dipole, passive, reciprocal and linear network. If fraction a1 of this power is absorbed by resistor R1, at temperature T1, fraction a2 of resistance R2 at temperature T2, fraction a3 by resistor R3 at temperature T3, etc., then the equivalent temperature of dipole noise (called and effective temperature) is given by the relation (4.7a), where we have the relation

(4.7b).

Modeling the dipole noise

- **Application**

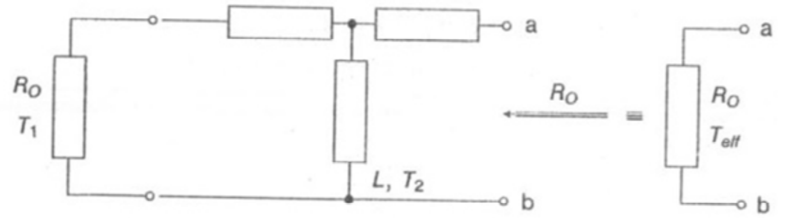


Fig.4.3

$$T_{eff} = a_1 T_1 + a_2 T_2 \quad \text{cu} \quad a_1 + a_2 = 1 \quad (4.8)$$

$$a_1 = \frac{1}{L} \quad (4.9)$$

$$a_2 = 1 - a_1 = 1 - \frac{1}{L} \quad (4.10)$$

$$T_{eff} = \frac{1}{L} T_1 + \left(1 - \frac{1}{L}\right) T_2 \, (4.11)$$

Statement

We consider a suitable attenuator, whose attenuation is denoted by L and the temperature by T2 (Fig. 4.3). If at the input a resistor R0 with temperature T1 is connected, what is the equivalent temperature of noise T at the output? What is the equivalent temperature of noise at the attenuator input?

Solution

The noise power generated by Ro is added to the noise power provided by the attenuator resistive network, as both are decorated. It is required to calculate the effective Teff temperature of the resistance R0 seen from the terminals (a-b), which produces the same noise power as the source circuit.
Pierce's rule is applied, inverting the transmission: we assume that a unitary power of noise is applied to terminals a-b. We consider that the fraction of this power is absorbed by R0 and the fraction of this power is absorbed by the attenuator. We have Eq. (4.8).
Since the attenuation introduced by the attenuator is L, it results that the fraction of the signal (unitary) that reaches the terminals of the resistance R0 is (4.9).
Therefore, the power fraction that is absorbed in the attenuator is (4.10).
The equivalent noise temperature at the attenuator output is (4.11)

Modeling the dipole noise

- **Application**

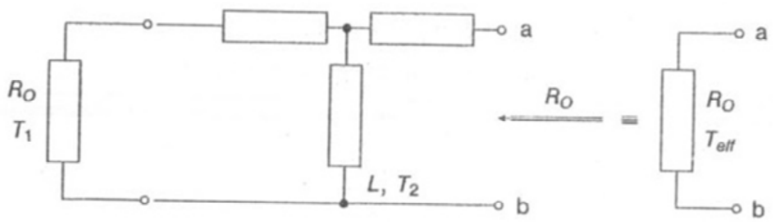


Fig.4.3

$$kT_{eff} = \frac{k}{L}\left(T_1 + (L-1)T_2\right) \quad (4.12)$$

$$N_o = kG_a\left(T_s + T_n/G_a\right)\delta f = kG_a\left(T_s + T_e\right)\delta f \quad (3.46b)$$

$$T_e = (L-1)T_2 \quad (4.13)$$

Eq. (4.11) can be presented in the form (4.12), which, by identification with (3.46b), leads to the equivalent temperature at the input of the matched attenuator, Eq. (4.13).

Conclusion

The noise representation of a passive, linear dipole type network has two aspects:
If all resistors are at the same temperature T, the network is characterized either by the Thevenin model ($\overline{(v_n ^ 2)} = 4kTR_{eq}\Delta f$), or the Norton model ($\overline{(i_n ^ 2)} = 4kT G_{eq}\Delta f$), or by the available noise power (P = kTΔf). In this case $T_{eff} = T$.
If the different resistors Rj are found at different temperatures Tj, the only difference is that in the models listed above, the temperature T is replaced with the effective Teff temperature calculated using Pierce's rule.

Modeling the noise of a quadripole

- **Noisy quadripol**

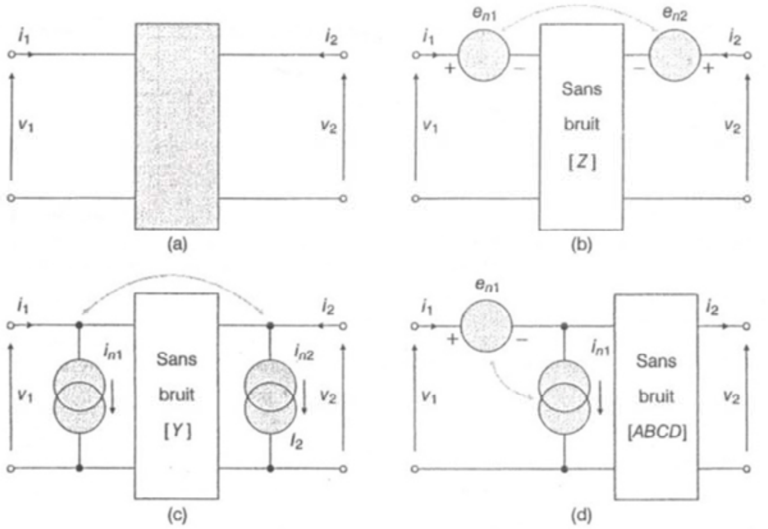


Fig.4.4

For modeling you will always need 6 parameters: 4 to characterize the ideal passive quadripole (noiseless) and 2 parameters to take into account the two equivalent sources of noise located at the gates (according to figures 4.4 b, c or d. These sources they are always partially correlated.

The problem

It is essential to find the correlation that exists between the two sources from the gates. This evaluation can be done in the temporal domain (with the help of the following two theorems) or in the frequency domain (in which case the two sources are characterized by their own spectral power densities, own and cross).

Modeling the noise of a quadripole

- **Montgomery's theorem 1**

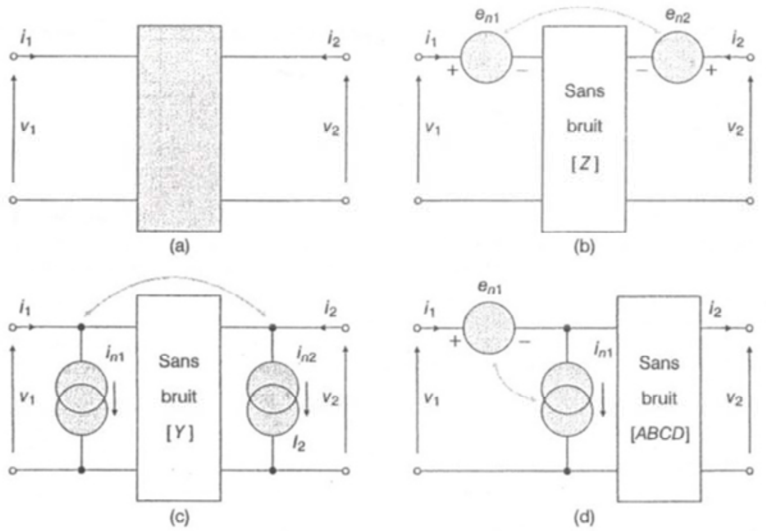
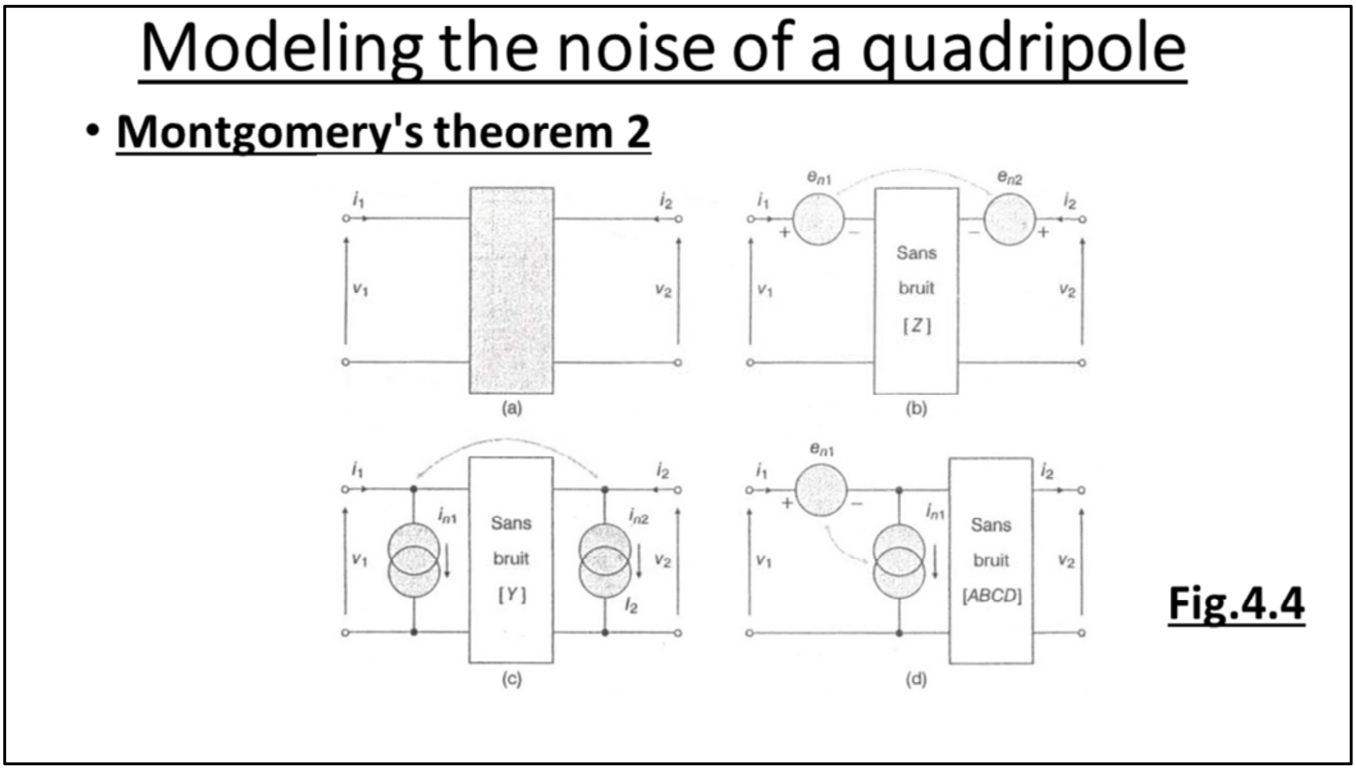


Fig.4.4

Montgomery's theorem 1

If two currents (or voltages) originate partly in a common source and partially in different sources, and if α is the power fraction transferred between the common source and the first noise current (voltage), while β is the power fraction of the common source transferred to that of the two current source (voltage) of noise, then the correlation coefficient that exists between the two currents (voltages) considered is the geometric mean between α and β.



Montgomery's theorem 2

The correlation coefficient between two noise currents (voltages) remains unchanged if one or both cross linear networks, characterized by real transfer functions.

Comment

The second theorem justifies the historical preference expressed for the scheme presented in Fig. 4.4d, where there is a separate net between the part of the circuit that contains the sources of noise and the ideal quadripole, presumably noisy. In this case, we only need to find the correlation coefficient at the entrance of the linear quadripole, which will be the same at the output.

Modeling the noise of a quadripole

- **Modeling in the time domain**
  - *The Rothe and Dahlke model*

$$e = e_n + e' \quad (4.14)$$

$$Z_{cor} = R_{cor} + jX_{cor} \quad (4.15)$$

$$e = e_n + iZ_{cor} \quad (4.16)$$

$$Y_{cor} = G_{cor} + jB_{cor} \quad (4.17)$$

$$i = i_n + eY_{cor} \quad (4.18)$$

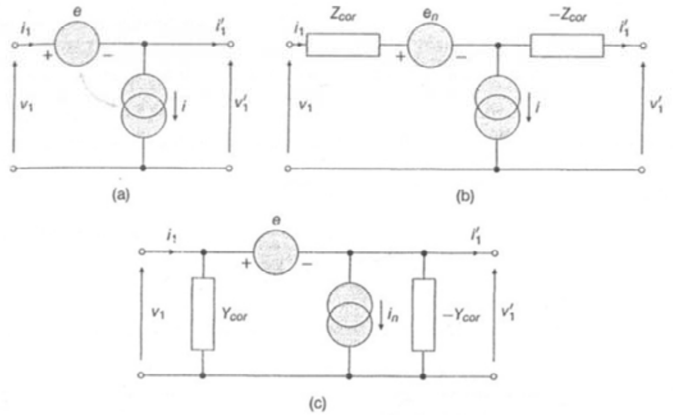


Fig.4.5

If we consider the representation of the cascade type, the two sources, denoted "e" and "I", constitute the quadripole of noise, (Fig.4.5a).
The noise voltage can be put in the form of a sum of two terms, eq. (4.14), where the first term en is assumed to be independent of I, while (e ') is totally correlated with i.
In order to study the circuit using conventional methods, Rothe and Dahlke proposed to replace the correlation coefficient of the generators with a correlation impedance, eq. (4.15), so that we have (4.16).

The dual reasoning (the decomposition of the current into two terms, leads to the introduction of the correlation admittance (4.17), which allows the relationship to be written (4.18).

Modeling the noise of a quadripole

- **Modeling in the time domain**
  - *The Rothe and Dahlke model*

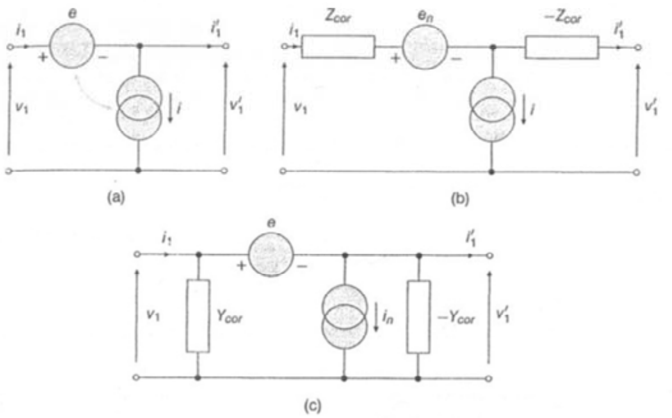


Fig.4.5

$$v_1 = v + v_1' = e_n + iZ_{cor} + v_1' = e_n + \left(i_1 - i_1'\right)Z_{cor} + v_1' \quad (4.19a)$$

$$i_1 = i + i_1' = i_n + eY_{cor} + i_1' = i_n + \left(v_1 - v_1'\right)Y_{cor} + i_1' \quad (4.19b)$$

The circuit in Fig.4.5a and equation (4.16) lead to (4.19a), or for the dual situation (4.19b).

Modeling the noise of a quadripole

- **Modeling in the time domain**
  - *The Rothe and Dahlke model*

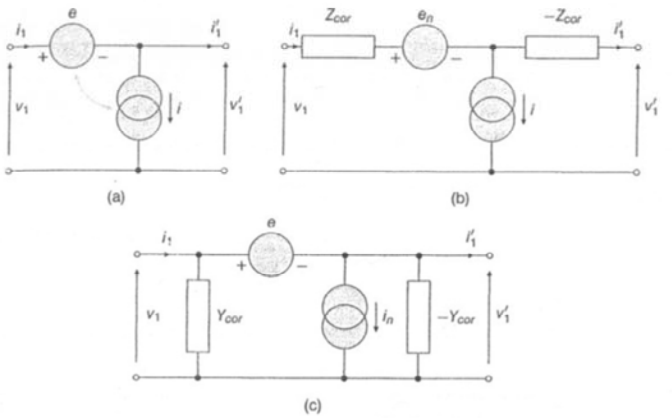


Fig.4.5

$$v_1 = v_1' + e_n + \left(i_1 - i_1'\right) Z_{cor} \quad (4.20a)$$

$$i_1 = i_1' + i_n + \left(v_1 - v_1'\right) Y_{cor} + \quad (4.20b)$$

Equations (4.19), put in the form (4.20) , lead to the equivalent noise circuits presented in (Fig.4.5b and c), which are Rothe and Dahlke's models.

Remarks
The imitances Zcor and Ycor are supposed to be ideal (no noise). This condition is often highlighted by writing T = 0 next to their symbol.
The existence of the imitances –Zcor and –Ycor is the consequence of the negative terms present in the equations (4.20); on the other hand, these negative imitators compensate for the positive ones, which ensures zero attenuation for the signals that cross the quadripoles illustrated in Fig.4.5b and c (because from the point of view of the signal, the noise generators are presumed ideal).

Modeling the noise of a quadripole

- **Modeling in the time domain**
  - *The Rothe and Dahlke model*
  - *Parameters of the model*

$$\overline{e^2} = 4kT_0R_n\Delta f, \quad \overline{i_n^2} = 4kT_0G_n\Delta f \quad (4.21a)$$

$$\overline{e_n^2} = 4kT_0r_n\Delta f, \quad \overline{i^2} = 4kT_0g_n\Delta f \quad (4.21b)$$

Tabel 4.1

$\Pi \Rightarrow T$	$T \Rightarrow \Pi$
$g_n = G_n + R_n\lvert Y_{cor}\rvert^2$	$R_n = r_n + g_n\lvert Z_{cor}\rvert^2$
$r_n = \dfrac{G_n}{\lvert Y_{cor}\rvert^2 + (G_n/R_n)}$	$G_n = \dfrac{r_n}{\lvert Z_{cor}\rvert^2 + (r_n/g_n)}$
$Z_{cor} = \dfrac{Y_{cor}^*}{\lvert Y_{cor}\rvert^2 + (G_n/R_n)}$	$Y_{cor} = \dfrac{Z_{cor}^*}{\lvert Z_{cor}\rvert^2 + (r_n/g_n)}$

Instead of the generators, for the model in Π the equivalent noise resistance Rn and the equivalent conductance Gn are introduced, using equations (4.21a), and for the T-scheme, we have (4.21b).
The noise behavior is thus described by a set of 3 parameters Rn, Gn and Ycor (or rn, gn and Zcor). These are bound by the relationships given in Table 4.1.

Characterization of the cuadripol
1) For any quadripole, the noise factor F varies with the signal source admittance and has a minimum, noted F0, called the minimum noise factor.
2) The particular value of the source admittance corresponding to this minimum is called the optimal source admittance and is noted with Yo = Go + jBo.
3) The set consisting of 4 parameters, Fo, Go, Bo and Rn completely characterize the noise behavior of the quadripole.

Modeling the noise of a quadripole

- **Modeling in the time domain** • *The Rothe and Dahlke model*
  - *Calculation of noise parameters*

$$F = 1 + \frac{G_n + R_n\left(\left(G_s + G_{cor}\right)^2 + \left(B_s + B_{cor}\right)^2\right)}{G_s} \quad (4.22a)$$

$$F = 1 + \frac{r_n + g_n\left(\left(R_s + R_{cor}\right)^2 + \left(X_s + X_{cor}\right)^2\right)}{R_s} \quad (4.22b)$$

In order to find the relationships between the classical noise parameters and the equivalent generators at the gates, the expression of the noise factor must first be established. Then, with respect to the source admittance, there is the minimum noise factor Fo and the optimal admittance Yo.
Thus, for the Rothe and Dahlke model (scheme in Π), using the definition of North, one obtains (4.22a), and for the scheme in T one obtains (4.22b)

Modeling the noise of a quadripole

- **Modeling in the time domain**
- *The Rothe and Dahlke model*
- *Calculation of noise parameters*

Tabelul 4.2

Schema in Π	Schema in T
$B_o = -B_{cor}$	$X_o = -X_{cor}$
$G_o = \sqrt{(G_n/R_n) + G_{cor}^2}$	$R_o = \sqrt{(r_n/g_n) + R_{cor}^2}$
$F_o = 1 + 2R_n(G_o + G_{cor})$	$F_o = 1 + 2g_n(R_o + R_{cor})$
$F = F_o + \frac{R_n}{G_s}\lvert Y_s - Y_o\rvert^2$	$F = F_o + \frac{g_n}{R_s}\lvert Z_s - Z_o\rvert^2$

Following the indicated steps, we find the results presented in Table 4.2

Modeling the noise of a quadripole

- **Modeling in the time domain** • *The Rothe and Dahlke model*
  - *Calculation of noise parameters*

$$G_{cor} = \frac{F_o - 1}{2R_n} - G_o, \; B_{cor} = -B_o, \; G_n = R_n\left(G_o^2 - G_{cor}^2\right) \quad (4.23)$$

The invers conversion is provided by Eqs. (4.23).

Conclusion

Rothe and Dahlke's model suppresses the correlation between generators in the equivalent scheme. The correlation coefficient is replaced by an impedance or an admittance of correlation (presumed without noise), which allows the analysis of the circuit by traditional methods. Another advantage of this model is that the electrical parameters of the quadripole do not interfere with the expressions of noise.

Modeling the noise of a quadripole

- **Relations between different parameters**

$$F = F_o + \frac{R_n}{G_s}\left[\left(G_s - G_o\right)^2 + \left(B_s - B_o\right)^2\right] \quad (4.24)$$

Substituting the relations (4.23) into the expression (4.22a) we obtain Eq. (4.24). This describes the variation of the noise factor in relation to the admittance of the signal source, at a constant frequency. It is called the "fundamental noise equation".

1) Fo is the minimum noise factor, which we can obtain for a quadripole perfectly adapting the signal source (Ys = Yo).
2) Rn is a passive parameter having the size of a resistance that "quantifies" to a certain extent the effect of mismatching (Ys ≠ Yo). In practice, it intervenes especially in the amplification circuits of broadband and low noise, where the mismatchings are inevitable, especially at the ends of the band. In this case, a low Fo must be accompanied by a small Rn.
3) Go and Bo are optimal values of the real and imaginary part of the Ys source admittance. They almost always differ from the values that lead to maximum power gain.

Remark
In practice, a matching circuit is introduced between the source and the quadripole to change the source impedance.
It should be noted that the minimum noise matching (Ys = Yo) does not imply the simultaneous adaptation from the point of view of the signal.

Modeling the noise of a quadripole

- **Lange's contribution**

$$N = R_n G_o \quad (4.25a)$$

$$F = F_o + N\frac{\left|Z_s - Z_o\right|^2}{R_s R_o} \text{ sau } F = F_o + N\frac{\left|Y_s - Y_o\right|^2}{G_s G_o} \quad (4.25b)$$

Lange defines Rn as a constant that shows how much the minimum noise factor deteriorates when the input is closed on a non-optimal admittance.
In order to facilitate the microwave characterization, he proposes to use instead of Rn the parameter N, defined in (4.25a), which leads to equivalent forms (4.25b).
The advantages offered by this new constant are the following.
1) The dual expressions (4.25b) are perfectly symmetrical.
2) N, like Fo, depends only on the internal transistor (chip) and not on the connections associated with the encapsulation (provided they are lossless).
3) In a measurement system with transmission lines, N does not depend on the position of the reference plane.
4) By connecting several identical devices in parallel, the values of N and Fo remain unchanged (which is interesting for the concept of devices having different active areas).

Modeling the noise of a quadripole

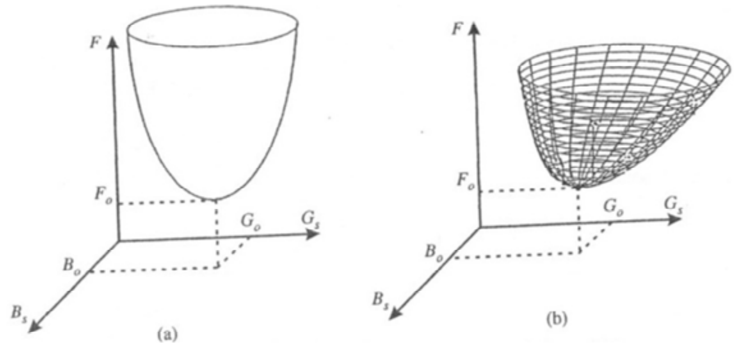
- **The relationship between the signal / noise ratio and F**

$$\frac{S}{N} = 10\log\left(\frac{V_s^2}{4kTR_s}\right) - F_{dB}\Big|_{\text{acelasi Rs}} \qquad (4.26)$$

The switch between the signal / noise ratio and the noise factor is made with Eq. (4.26), where Vs is the actual value of the signal source and Rs is the internal resistance..

Modeling the noise of a quadripole

- **Noise surfaces**



If the quadrupole studied is a transistor, it is interesting to represent the surface defined by the fundamental noise equation (4.24).

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **The noise matrix**

$$\mathbf{N} = \langle \mathbf{SS}^+ \rangle = \left\langle \begin{bmatrix} S_1 \\ S_2 \end{bmatrix} \begin{bmatrix} S_1^* & S_2^* \end{bmatrix} \right\rangle = \begin{bmatrix} \langle S_1 S_1^* \rangle & \langle S_1 S_2^* \rangle \\ \langle S_2 S_1^* \rangle & \langle S_2 S_2^* \rangle \end{bmatrix} \quad (4.27)$$

$$\langle VI^* \rangle = 4\pi \Delta f S_\omega (iv) = 2\Delta f \, S_f (iv) \quad (4.28)$$

$$\langle S_i S_j^* \rangle = \overline{S_i S_j^*} = 2\Delta f \, S_f (ji) \quad (4.29)$$

In this case, the sources of noise are described by their average powers in the band Δf centered on the frequency f. sources of noise with its adjoint vector (the adjoint matrix is obtained as follows: 1) each element is replaced by the complex conjugate; 2) the matrix thus obtained is transposed) and then considering the average values, Eq. (4.27).
Haus and Adles established that the average complex fluctuations of the cross product between voltage v and current I depend on the spectral density of cross power after Eq. (4.28).
Thus, in general, for two fluctuating quantities (denoted Si and Sj) where in addition the statistical mean is equal to the temporal average, one can write Eq. (4.29).
Factor 2 is justified by the domain of power calculation, which is traditional (within signal processing) between-) and + ∞, while in noise theory the power spectral density is defined only for positive frequencies.

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **The correlation matrix**

$$\mathbf{C} = \frac{1}{2\Delta f}\mathbf{N} = \frac{1}{2\Delta f}\begin{bmatrix} \langle S_1 S_1^* \rangle & \langle S_1 S_2^* \rangle \\ \langle S_2 S_1^* \rangle & \langle S_2 S_2^* \rangle \end{bmatrix} \quad (4.30)$$

For two sources of some noise, denoted S1 and S2, the correlation matrix is calculated using the relation (4.29), where S1 and S2 are the equivalent noise generators of the adopted representation, Eq. (4.30).

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **Particular cases**

$$\mathbf{C}_Z = 2kT\Re\{\mathbf{Z}\} \quad (4.31a)$$

$$\mathbf{C}_Y = 2kT\Re\{\mathbf{Y}\} \quad (4.31a)$$

In the case of a passive quadripole, which only generates thermal noise. The correlation matrices, depending on the open circuit impedance matrix Z or the short circuit admittance matrix Y, are Eq. (4.31a, b)
If the quadripole is reduced to an elemental structure consisting of a single resistance in series or a single conductor in parallel, the expressions (4.31) give the possibility to reconstruct Nyquist's formulas for thermal noise.

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **Chain representation**

$$\mathbf{N}=\begin{bmatrix}\langle e_n e_n^*\rangle & \langle e_n i_n^*\rangle \\ \langle i_n e_n^*\rangle & \langle i_n i_n^*\rangle\end{bmatrix}=2\Delta f\mathbf{C}_A \quad (4.32a)$$

$$\mathbf{C}_A=2kT\mathbf{C}_A^0 \quad (4.32b)$$

$$\mathbf{C}_A^0=\begin{bmatrix}C_{A11} & C_{A12} \\ C_{A21} & C_{A22}\end{bmatrix}=\begin{bmatrix}R_n & \dfrac{F_o-1}{2}-R_nY_o^* \\ \dfrac{F_o-1}{2}-R_nY_o & R_n|Y_o|^2\end{bmatrix} \quad (4.32c)$$

The noise matrix is (4.32a), with the notations in the figure (4.4d) and considering CA as the correlation matrix. This is given by the relation (4.32b), where CA0 is the normalized correlation matrix (4.32c).

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **Chain representation**

$$R_n = C_{A11} \quad (4.33a)$$

$$Y_o = G_o + jB_o = \sqrt{\frac{C_{A22}}{C_{A11}} - \left\{\Im\left(\frac{C_{A12}}{C_{A11}}\right)\right\}^2} + j\Im\left(\frac{C_{A12}}{C_{A11}}\right) \quad (4.33b)$$

$$F_o = 1 + 2\left(\Re\left(C_{A12}\right) + C_{A11}G_o\right) \quad (4.33c)$$

$$F = 1 + 2\Re\left(\frac{C_{A11}}{R_S} + C_{A12} + C_{A21} + C_{A22}R_S\right) \quad (4.33d)$$

The calculation of the noise parameters is performed with Eqs. (4.33)

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **Admitance representation**

$$\mathbf{N} = \begin{bmatrix} \langle i_{n1} i_{n1}^* \rangle & \langle i_{n1} i_{n2}^* \rangle \\ \langle i_{n2} i_{n1}^* \rangle & \langle i_{n2} i_{n2}^* \rangle \end{bmatrix} = 2\Delta f \mathbf{C}_I \quad (4.34a)$$

$$\mathbf{C}_I = 2kT\mathbf{C}_I^0 \quad (4.34b)$$

$$\mathbf{C}_I^0 = \begin{bmatrix} C_{I11} & C_{I12} \\ C_{I21} & C_{I22} \end{bmatrix} = \begin{bmatrix} G_n + |y_{11} - Y_{cor}|^2 R_n & y_{21}^* (y_{11} - Y_{cor}) R_n \\ y_{21} (y_{11} - Y_{cor})^* R_n & R_n |y_{21}|^2 \end{bmatrix} \quad (4.34c)$$

In this case, the noise matrix is (4.34a). The correlation matrix is (4.34b), where the normalized correlation matrix is in Eq. (4.34c)

Modeling the noise of a quadripole

- **Modelling in the frequency domain**
- **Admitance representation**

$$R_n = \frac{1}{|y_{21}|^2} \frac{\langle i_{n2} i_{n2}^* \rangle}{4kT_0 \Delta f} = \frac{1}{|y_{21}|^2} C_{I22} \quad (4.35a)$$

$$Y_{cor} = G_{cor} + jB_{cor} = y_{11} - y_{21} \frac{\langle i_1 i_2^* \rangle}{\langle i_2 i_2^* \rangle} = y_{11} - y_{21} \frac{C_{I12}}{C_{I22}} \quad (4.35b)$$

$$G_n = \frac{\langle I_1 I_1^* \rangle}{4kT_0 \Delta f} - \frac{\langle I_1 I_2^* \rangle}{\langle I_2 I_2^* \rangle} \frac{\langle I_1^* I_2 \rangle}{4kT_0 \Delta f} = C_{I11} - \frac{C_{I12}}{C_{I22}} C_{I21} \quad (4.35c)$$

$$4kT_0 \Delta f (F-1) = \langle I_{n1} I_{n1}^* \rangle + \left| \frac{y_{11} + Y_S}{y_{21}} \right|^2 \langle I_{n2} I_{n2}^* \rangle - 2\Re \left(\frac{y_{11} + Y_S}{y_{21}} \langle I_{n2} I_{n1}^* \rangle \right) \quad (4.35d)$$

The calculation of the noise parameters is performed with Eqs. (4.35a-c), to which we add the relations from Table (4.2) (diagram in Π).
The noise factor is calculated with the relation (4.35d).