NOISE AND DISTURBANCES

Ch.3
Noise parameters

Introduction

$$\overline{v_n^2} = 4kTR\Delta f$$

$$\overline{i_n^2} = 4kT\Delta f / R$$

In Chapter 2 we saw how to represent the noise produced by a resistor, either in the form of a Thevenin generator or as a Norton generator.

In practice, it is convenient to represent the other categories of noise by converting to equivalent thermal noise. This operation has two possibilities:
In the slide equations we keep T equal to the physical temperature of the system and modify R or G to obtain the same noise power as the studied source. Thus the notions of resistance (or conductance) of noise equivalent are introduced.
We keep R or G to their physical value, and adjust T. In this case we reach the notion of equivalent noise temperature.
Such a description requires us to ensure in advance that the spectrum of the modelled source is of the white type, as is the thermal noise.

Another approach is to represent noise using spectral densities. This method is less intuitive, even uncomfortable, because of the extremely small numerical values associated with the generators.

For this reason we prefer in practice the use of parametric parameters such as equivalent noise resistance, noise temperature or noise factor, because they are much more intuitive, have common numerical values and are especially closely related to measurements.

In this case, we face two types of difficulties:

For each size there are several definitions (often equivalent, but under certain conditions, which are not always specified)
All noise parameters have a very high frequency dependence. Therefore, they are specified at a fixed frequency or by values in a broadband (when calling a statistical spread).

Notions of circuit theory

- **Powers defined in time domain**
  - Instant power

$$p(t) = v(t)i(t) \quad (3.1)$$

Example

$$v(t) = V\sqrt{2}\cos(\omega t + \theta) \text{ si } i(t) = I\sqrt{2}\cos(\omega t + \theta - \phi)$$

$$p(t) = VI\cos\phi + VI(2\omega t + 2\theta - \phi) \quad (3.2)$$

Notions of circuit theory

- **Powers defined in time domain**
  - **Active power**

$$P_{act} = \frac{1}{T}\int_0^T p(t)\,dt = VI\cos\phi \quad (3.3)$$

Active power is by definition the average value of instantaneous power.

Notions of circuit theory

- **Powers defined in time domain**
  - **The fluctuating power**

$$p_f(t) = VI\cos(2\omega t + 2\theta - \phi) \quad (3.4)$$

The fluctuating power is the sinusoidal quantity, of 2ω pulsation, from the Expression (3.2)

Notions of circuit theory

- **Powers defined in time domain**
- **Reactive power**

$$Q = VI\sin(\phi) \quad (3.5)$$

Notions of circuit theory

- **Powers defined in time domain**
- **The apparent power**

$$S = VI = \sqrt{P^2 + Q^2} \quad (3.6)$$

The apparent power is the maximum value of the active power (for cosø = 1).

Notions of circuit theory

- **Powers defined in frequency domain**
- **Complex power**

$$S = VI^* \quad (3.7)$$

$$S = ZII^* = Z\left|I\right|^2 \quad (3.8a)$$

$$S = VY^*V^* = Y^*\left|V\right|^2 \quad (3.8b)$$

Complex power is the product of the complex voltage, denoted by V, and the complex conjugate value of current I.

Notions of circuit theory

- **Powers defined in frequency domain**
- **Avarage power**

$$P_m = \Re\left(VI^*\right) = \frac{1}{2}\left(VI^* + V^*I\right) \quad (3.9)$$

Notions of circuit theory

- **Powers defined in frequency domain**
- **The average power of an anharmonic signal**

$$P_m = \left\langle I_n(j\omega) I_n(-j\omega) \right\rangle \quad (3.10)$$

$$S(P_m) = \left\langle I_n(j\omega) I_n(-j\omega) \right\rangle = \overline{I_n^2} \quad (3.11)$$

$$S(P_m) = \overline{I_n^2} = 4kTG \quad (3.12)$$

We consider a noise current In, which could be expressed as an infinite amount of harmonic currents, all having multiple frequencies of fundamental.
In this case, the average power is defined by the total current In, Eq. (3.10). This represents the average normalized power developed in a unit band.
The expression (3.10) is the normalized average power spectral density, Eq. (3.11).
In the case of the thermal noise generated by a conductor G, we have (3.12)

Notions of circuit theory

- **Available power and available power gain**

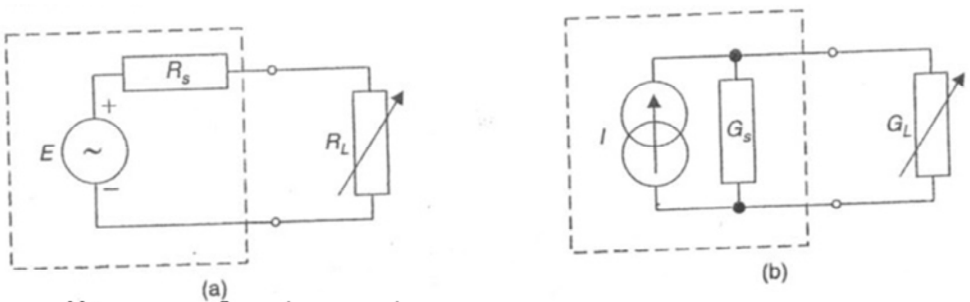


Fig.3.1

$$P_L = \frac{R_L}{\left(R_L + R_S\right)^2} E^2 \quad (3.13a)$$

$$P_a = \max\left(P_L\right) = \frac{E^2}{4R_S} = \frac{I^2}{4G_S} \quad (3.13b)$$

The available power is defined as the maximum power transferred by a dipole to the load, when the load is assumed to be adjustable.
We consider an active dipole of internal resistance Rs, which flows on an RL load, represented by the equivalent scheme Thevenin (fig.3.1a) or Norton (fig.3.1b). The power dissipated in the load is Eq. (3.13a).
If Rs = RL, the load is said to be adapted to the source and half of the total power of the generator goes to load, and half dissipates on its own internal resistance. In this case, the power transferred to the load is maximum. And it's called available power. The available power is noted with Pa and is given by Eq. (3.13b)

Notions of circuit theory

- **Generalization to a general dipole**

Matching condition

$$Z_L = Z_S^*$$

$$P_a = \frac{EE^*}{4R} = \frac{EE^*}{2\left(Z+Z^*\right)}, \quad \text{pentru R>0} \quad (3.14)$$

$$P_a = \frac{\overline{EE^*}}{4R} = \frac{\overline{EE^*}}{2\left(Z+Z^*\right)} \quad (3.15)$$

The expression (3.14) is deduced by assuming that the signal delivered by the dipole is harmonic and of complex voltage E.

If the signal delivered by the source is a noise, Eq. (3.14) needs to be considered the average value of the product, Eq. (3.15)

Notions of circuit theory

- **Particular cases**

The case of a resistance

$$P_a = \frac{4kTR\Delta f}{4R} = kT\Delta f \;\; [W] \quad (3.16)$$

The case of a diode

$$P_a = \frac{kT\Delta f}{2} \quad (3.17)$$

The case of resistance
A resistor produces a thermal noise whose available power is Eq. (3.16)

The case of a diode
A diode having a load resistance adapted to the differential conductor, has an available noise power given by Eq. (3.17).

Notions of circuit theory

- **Gain in available power**

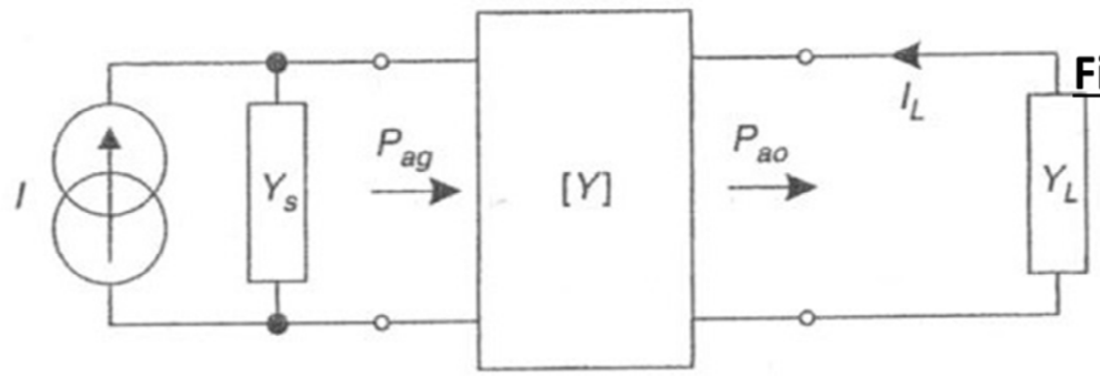


Fig.3.2

$$G_a = \frac{P_{ao}}{P_{ag}} \quad (3.18)$$

We consider an active quadripole interspersed between the internal admittance signal generator Ys and the load YL, Fig. 3.2.
The gain in available power of the quadripole, denoted Ga, is defined as the ratio between the power available at the output Pao and the available power of the Pag generator, Eq. (3.18).

Notions of circuit theory

- **The case of a resistive quadrupole interspersed in a resistive circuit**

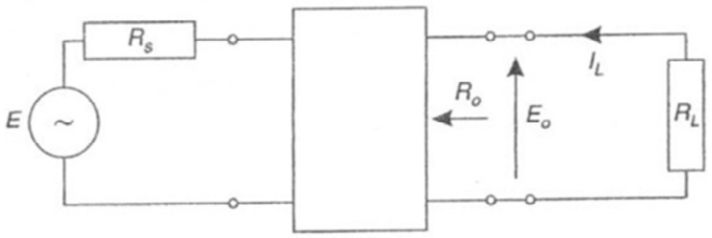


Fig.3.3

$$G_a = \frac{E_o^2/4R_o}{E^2/4R_S} = \frac{E_o^2 R_S}{E^2 R_o} = A_v^2 \frac{R_S}{R_o} \quad (3.19)$$

$$N_o = \frac{E_o^2}{4R_o} = \frac{(A_v v_n)^2}{4R_o} = \frac{A_v^2 4kTR_S\Delta f}{4R_o} = G_a kT\Delta f \quad (3.20)$$

Let be a sinusoidal generator of internal resistance Rs, and of electromotor voltage E which supplies a quadripole which presents at its output a resistance Ro and voltage Eo, fig.3.3.
The gain in available power is (3.19), where Av is the voltage gain, defined in relation to the generator. Eq. (3.19) shows that the gain in available power does not depend on the load, but only on the internal resistance of the generator and how the generator is coupled to the load. Therefore Ga is not a characteristic of quadripole, because he also considers the generator.

If we now assume that the input generator is a noise generator in association with the resistor Rs, the relation (3.19) remains valid, the power available at the output being this time the noise power No, Eq. (3.20).

It should be noted that the gain in available power remains valid regardless of the type of signal; for this reason this gain is suitable for noise calculations.

Notions of circuit theory

- **The case of several floors in the waterfall**

$$\left(G_a\right)_t = G_{a1}G_{a2}\ldots G_{aN} = \frac{P_1}{P_a}\frac{P_2}{P_1}\cdots\frac{P_N}{P_{N-1}} = \frac{P_N}{P_a} \quad (3.21)$$

Pk is the power available at the output of quadripole k and Pa represents the available power of the generator.

Notions of circuit theory

- **Exchangeable power**

$$P_a = \frac{1}{4}\frac{I \cdot I^*}{\Re(Y_S)} \quad pentru \Re(Y_S) > 0 \quad (3.21a)$$

$$P_e = \frac{1}{4}\frac{I \cdot I^*}{\Re(Y_S)} \quad pentru \Re(Y_S) < 0 \quad (3.21b)$$

Let a sinusoidal signal generator, which is modeled by a YS admittance, parallel to a complex value current source I. The available power, after the relation (3.14) is Eq. (3.21a).
If Re (Ys) <0, the interchangeable power is introduced. As the maximum (stationary) value of the power flow, which enters or exits a gate, obtained for an arbitrary variation of the voltage or current of this gate, Eq. (3.21b).
If Re (Ys)> 0, the value of Pe is positive and it is identified with the available power. If Re (Ys) <0, the value of Pe is negative and it represents the power extracted by an adapted load Y_S^*.

Notions of circuit theory

- **Equivalent noise band**

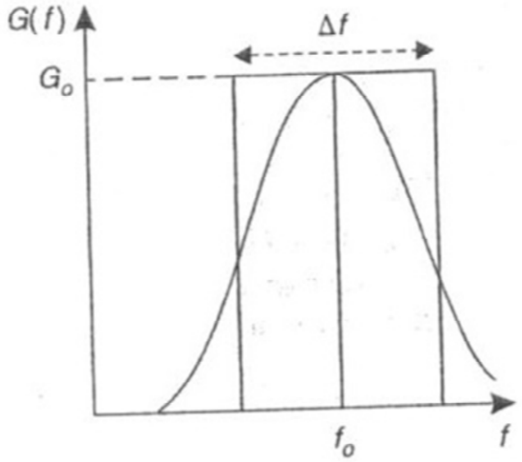


Fig.3.4

$$f_0 = \sqrt{f_b f_h} \quad (3.22)$$

$$f_0 \simeq (f_b + f_h)/2 \quad (3.23)$$

$$P_{tot} = G_0 kT \Delta f = kT \int_0^{+\infty} G(f)\,df \quad (3.24)$$

$$\Delta f = \frac{1}{G_0} \int_0^{+\infty} G(f)\,df \quad (3.25)$$

Because noise is the result of a lot of random signals, we are forced to introduce another definition for the equivalent noise band, which is necessarily different from the classical band defined at 3 dB, in the case of harmonic signals.

The signal Bandpass

For an amplifier with a given circuit, the bandwidth B is defined as the frequency range between the points where the output power decreases by half of its maximum value (B = fh-fb). This halving of the power corresponds to the reduction of the output voltage by 70.7% (or 3 dB).

In general, the given circuits have a characteristic of symmetrical frequency: the central frequency f0 is the geometric mean of the cutting frequencies, Eq. (3.22).

For selective circuits with high quality factor, we can approximate the central frequency by the arithmetic mean of the cutting frequencies, Eq. (3.23).

Equivalent noise band.

Noted Δf, the equivalent noise band is, by definition, the band of an ideal circuit (having the power characteristic transmitted according to the frequency of rectangular form), which lets the same noise power pass as the real circuit. This situation is illustrated in Fig. 3.4, where G (f) represents the real characteristic of the gain in power, with the same surface as the rectangle of height G0 and width Δf, which corresponds to the ideal circuit.

In this way, we ensure the equality between the total noise power transmitted inside the band Δf and the total noise power that passes through the real circuit. If the input noise is thermal and if the circuit does not add noise, we can write the relation (3.24). , from where the relationship results (3.25).

The gain in power considered by definition is the gain in available power.
Eq. (3.25) is in line with the tradition that the equivalent noise band is "one-sided".
The fact that the equivalent is made with a rectangle implies implicitly that we have adopted only white noise.
If the gain is described by a transfer function having the cutoff frequency fc, the equivalent noise band is (π / 2) fc.

Notions of circuit theory

- **Equivalent noise band**

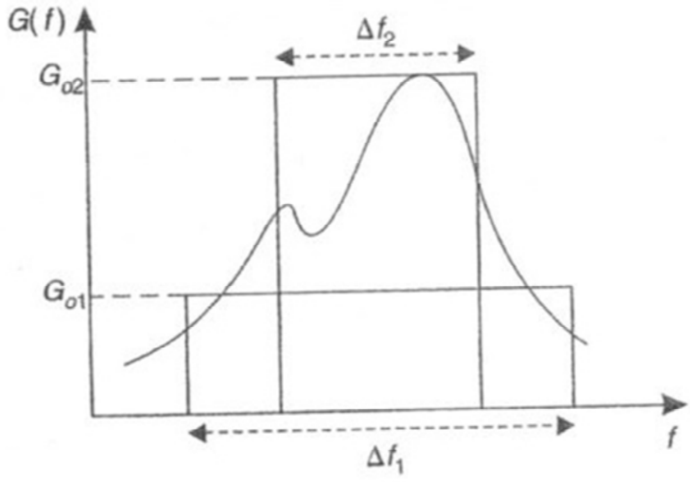


Fig.3.5

If the real characteristic G (f) is irregular, as in Fig.3.5, it is difficult to define the central frequency (where the height of the rectangle is normally established).
There are several possibilities, of which two are illustrated in Fig. (3.5), to determine the equivalent band Δf1 or Δf2 depending on the choice of the “maximum” gain (G01 or G02).

Notions of circuit theory

- **Method of evaluation**

$$\Delta f = \frac{1}{A_{v0}^2} \int_0^{+\infty} \left| A_v(f)^2 \right| df \qquad (3.26)$$

In practice we usually record the variation of the gain in voltage Av in relation to the frequency; assuming that its maximum is Av0. Since the gain in power is proportional to the square of the gain in voltage, we see Eq. (3.26).

Notions of circuit theory

- **Relationship with the bandpass**

$$\Delta f = \frac{B}{\sqrt{2^{1/n}-1}}\int_0^\infty \left(\frac{1}{1+x^{2n}}\right)^m dx \quad (3.27)$$

In practice, the band at 3dB of a system (denoted by B) is well known and not the equivalent noise band Δf, which is why we often tend to consider Δf = B.

Assuming that the system consists of identical stages in cascade, each having distinct poles, then the equivalent noise band can be calculated with Eq.3.27.

It is found that as the number of stages (or the number of poles) increases, the more the difference between the equivalent band of noise and the band at 3 dB, decreases.

Noise of a dipole
Narrow band parameters

- **Relationship with the bandpass**

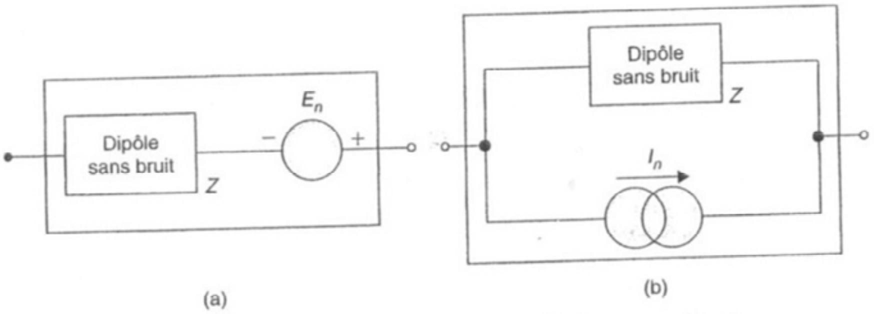


Fig.3.6

$$S(E_n) = |Z|^2 \, S(I_n) \quad (3.28)$$

$$E_n = Z \cdot I_n \quad (3.29)$$

Peliminary remark
The narrow band noise parameters of a dipole or quadripole are defined for an elementary band, which is normally 1 Hz.
For tradition's sake, the introduction of definitions uses powers instead of spectral power densities. This requires the use of a band that is not unitary but small enough to respect the "narrow band" spiritually. This band will be denoted by δf, but it will not appear in the final expression of the parameter.

Modeling
The noise of a dipole is represented either by an ideal voltage generator, En, placed in series with the assumed noise-free dipole (Thevenin’s model), or by an ideal current generator, In, placed parallel to the assumed noise-free dipole (the Norton’s model).

Noise of a dipole
Narrow band parameters

- **Equivalent noise resistance**

$$R_n = \overline{v_n^2} / 4kT_0\Delta f \quad (3.30)$$

$$R_n = \overline{v_n^2} / 4kT\Delta f \quad (3.31)$$

$$R_n = \pi S_v / kT_0 \quad (3.32)$$

$$R_n = \overline{v_n^2} / 4kT_0\Delta f \quad (3.33)$$

Nielsen's definition

If a noisy dipole is represented by the Thevenin equivalent circuit, then the equivalent noise resistance is the value of a hypothetical resistor that, maintained at the reference temperature T0 = 290 K, produces the same (thermal) noise as the real dipole, Eq. (3.30).

Definition of Savelli

The equivalent noise resistance of a dipole is the fictitious resistance that, brought at the same temperature with the dipole, presents at its terminals the same quadratic mean value of the noise voltage as that of the real dipole terminals, Eq. (3.31).

Definition of IEEE

Equivalent noise resistance is a quantitative representation, in units of resistance, of the spectral density Sv of a noise voltage generator, at a specified frequency.
The relation between the equivalent noise resistance and the spectral density Sv of the generator is Eq. (3.32), where T0 = 290 K.
Equivalent noise resistance relative to the mean quadratic value, $\overline{(v_n^2)}$, within a frequency range Δf, ste Eq. (3.33), which returns to the first definition.

Remarks

If the dipole is itself at a reference temperature of 290 K, then the first and second

definitions coincide.
Van der Ziel adopts the expression (3.30) without imposing the value of the reference temperature.
No definition implies a physical resistance, placed somewhere inside the dipole, of Rn value.
Most likely, in the expression (3.32) Sv represents the bilateral spectral density, relative to ω.

The concept of equivalent noise resistance requires, implicitly, that the source of noise thus characterized, to provide white noise. If this is not the case then the use of the notion is questionable.

Noise of a dipole
Narrow band parameters

- **Equivalent noise current**

$$\overline{i_n^2} = 2qI_D\Delta f \quad (3.34)$$

$$I_n = \overline{i^2}/2q\Delta f \quad (3.35)$$

$$I_n = (2\pi S_i)/q \quad (3.36)$$

The average quadratic value of the noise current of a vacuum diode, saturated, is given by the relation (3.34), where ID is the diode current in the direct sense.

Definition of Van der Ziel
The equivalent saturated diode current is defined as the current of a saturated diode that produces a noise having the same spectral density as the current of the source considered. If the considered source produces a current having the mean quadratic value (i ^ 2)̅, the current equivalent current of the saturated diode is Eq. (3.35).

Definition of IEEE
The equivalent current of the saturated diode is a quantitative representation, in units of current, of the spectral density of the noise current generator, at a specified frequency. Particular forms: the relation between the equivalent noise current In and the spectral density Si, of the noise current generator is Eq. (3.36).

Noise of a dipole
Narrow band parameters

- **Noise temperature**

$$T = \frac{P_e/\Delta f}{k} = 7.25 \cdot 10^{+22} \frac{P_e}{\Delta f} \quad (3.37)$$

The available noise power produced by a resistor does not depend on its value and is given by the product kTΔf (eq.3.16). This expression suggests that the noise of a dipole can be characterized with the help of temperature T, whatever the origin of its noise (thermal or not).

Benett's definition
We introduce noise temperatures at a gate (of a dipole, quadripole, etc.), at a specified frequency, as the temperature of a passive system that provides an available power of noise, in a unit band, equal to that produced at the gate considered. .

We assume that the temperature is uniform throughout the passive system.
For a simple resistance, the noise temperature is equal to the actual temperature it is at, while for a diode the noise temperature may be different from the physical temperature. Noise temperature is a parameter that depends on the frequency.

The standard reference temperature adopted in all measurements is T0 = 290 K, for which kT / q = 25 mV.

Definition of Savelli
The noise temperature is defined as the Teq temperature at which the dummy dipole should be brought (with exclusive noise of thermal origin) so that it presents a noise

identical to that of the studied dipole, at temperature T, in the frequency range gamaf.

Definition of IEEE

It introduces noise temperatures measured in Kelvin degrees, at a gate, as the value of the ratio between the density of the exchangeable power and the Boltzmann constant, at the considered frequency.

The noise temperature T retains the sign of Re (Z), Z being the input impedance seen at the gate considered; for a gate to which a negative resistance is seen, it turns out that T is also negative.

Noise of a dipole
Narrow band parameters

- **Noise temperature of interconnected dipoles**

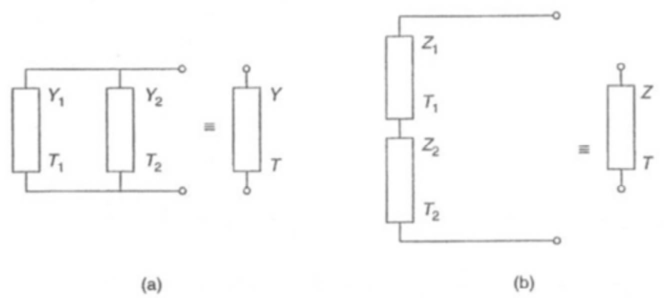


Fig.3.7

$$T = \frac{G_1T_1 + G_2T_2}{G_1 + G_2} \quad (3.38) \qquad T = \frac{R_1T_1 + R_2T_2}{R_1 + R_2} \quad (3.39) \qquad \frac{1}{T_1} < \frac{1}{T} < \frac{1}{T_2} \quad (3.40)$$

We consider a dipole consisting of the group of several independent dipoles. Each one is characterized by its noise temperature. To find the noise temperature of the equivalent dipole, we make the condition that the power output noise, in a band of 1 Hz, by the dipole and its equivalent, be equal.

For the case of a parallel friction (Fig. 3.7a), we have

4kT1G1 + 4kT2G2 = 4kTG, where Gi = re (Yi), I = 1.2.

Since G = G1 + G2,, we have Eq. (3.38)

For the case of serial connection (Fig.3.7b) we have

4kT1R1 + 4kT2R2 = 4kTR, with Ri = Re (Day), i = 1.2

Since R = R1 + R2, the result of Eq. (3.39)

In all cases, and whatever the sign of T1, T2 and T, Eq can be proved. (3.40)

Noise of a dipole
Narrow band parameters

- **Noise ratio**

$$N = \frac{\overline{E^2}}{4kTR\Delta f} = \frac{\overline{I^2}}{4kTG\Delta f} = \frac{R_n}{R} \quad (3.41)$$

$$N = \frac{T}{T_r} = \frac{S_p}{kT_r} \quad (3.42)$$

Savelli’s Definition

The noise ratio N of a dipole is the ratio between the spectral density (or the quadratic mean value) of the dipole noise generator and the same amount relative to a fictitious dipole, equivalent to the impedance dipole, whose only noise source would be of pure thermal origin, Eq. (3.41), where R and G are the real part of the impedance Z, or to the admittance Y, of the dipole.

Benett's definition

The noise ratio is the ratio of the equivalent temperature T, to the actual temperature Tr of the dipole, assumed in thermal equilibrium, Eq. (3.42), where Sp is the spectral power density measured in kT units.

Remarks

If the dipole is resistive, then N = 1 and the thermal noise has not been increased by the dipole.

N is usually expressed in decibels (considering 20 times the decimal logarithm of the ratio).

The quantity (N-1) measures the excess noise in relation to the thermal noise, long as the thermal noise unit.

Noise of a dipole
Narrow band parameters

- **The signal / noise ratio of a generator dipole**

$$\frac{S}{N} = \frac{P_{es}}{P_{en}} \quad (3.43)$$

The signal / noise ratio obtained in an ideal dipole (no noise) Z ‘, placed at the terminals of a generator, is equal to the ratio of the generator's exchangeable signal and noise powers, Eq. (3.43), whatever the impedance on which we measure the signal and the noise.

The noise of a quadripole

- **Equivalent Noise to the input**

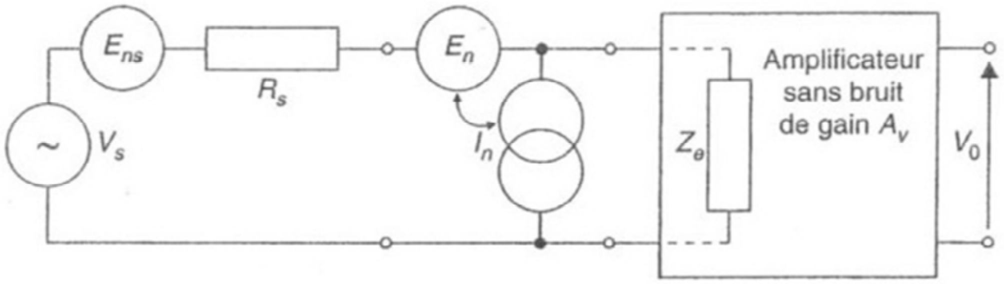


Fig.3.8

$$\overline{E_{ni}^2} = \overline{E_{ns}^2} + \overline{E_n^2} + \overline{I_n^2} R_s + 2C\overline{E_n I_n} R_s \quad (3.44)$$

The internal noise of an amplifier is often represented in the form of two correlated En and In noise generators, connected to the input (Fig. 3.8). I noted by the index "s" the parameters of the generator from the input, highlighting its thermal noise.

<u>The problem</u>
Given the input generator and amplifier described by its parameters, we are looking for the noise source Eni (called equivalent noise voltage at the input) which, placed at the level of Vs, can replace (in terms of effects) all three sources of noise. Ens, In, In.

<u>The solution</u>
Recalling that the In generator represents the noise supplied externally by the amplifier (so it cannot travel through Ze), the expression of the equivalent noise at the input is Eq. (3.44), where C is the correlation coefficient between the noise sources En and In.

<u>Equivalent noise resistance</u>
According to IEEE, this represents the value of the resistance, which applied to the input of an ideal amplifier (presumably noise-free, but having the same gain and the same band as the real amplifier) produces at the output the same noise, at T = 290 K. Remember that the noise spectrum delivered by the amplifier is white (which is rarely the case!).

The noise of a quadripole

- **Signal / noise ratio**

$$\frac{S}{N} = 10\log\left(\frac{V_s^2}{\overline{v_n^2}}\right) \quad (3.45)$$

It is desirable to obtain a higher signal / noise ratio at the output of a receiver. However, it is obvious that a poor signal-to-noise ratio does not necessarily mean poor receiver quality, as it is possible that the signal captured by the antenna may already have a poor signal-to-noise ratio.

Benett's definition

The signal-to-noise ratio is introduced as the ratio between the signal power and the noise power, at a precise location of the circuit, at a chosen frequency, Eq. (3.45).

Remark

This ratio is defined either across the band or at a specific frequency. In the first case, it is expressed in relation to the total noise N; In the second case, it is expressed in relation to the spectral density, which leads to a normalized signal / noise ratio. It is important to specify the signal band as the signal / noise ratio falls when the computation band increases beyond the signal band (because the circuit continues to add noise, while the signal power is constant).

Definition of IEEE

The signal / noise ratio is the ratio between signal and noise, both sizes being expressed identically (amplitudes, effective values). For example, the signal-to-noise ratio is often calculated as a ratio between the amplitudes of the useful signal and the noise superimposed on it.

Benett's definition is preferable because it is in line with the definition of the noise factor (which as we will see involves power ratio).

The noise of a quadripole

- **Equivalent noise temperature at the input**

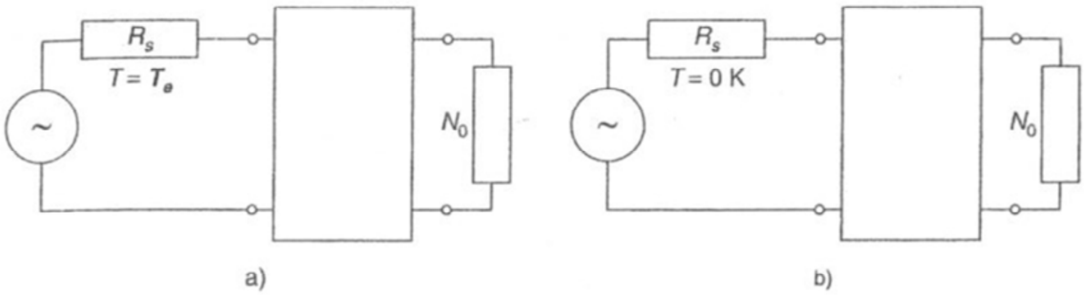


Fig.3.9

Let a quadripole fed at the input by a generator dipole of frequency fs and of internal resistance Rs. It is called the equivalent noise temperature of the input (expressed in Kelvins), the temperature Te that the generator dipole should have to produce at the output of the ideally assumed quadripole (Fig. 3.9a) the same noise power available as the real quadripole excited at the input by an ideal generator dipole (Fig.3.9b). By "ideal" is meant a quadripole or dipole noiseless, but with the same physical structure as the real circuit.

The noise of a quadripole

- **Equivalent noise temperature at the input**

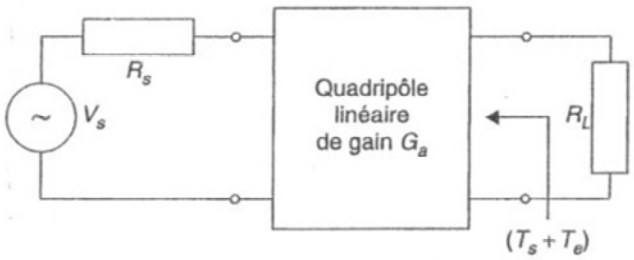


Fig.3.10

$$N_o = (kT_s\delta f)G_a + N_n = (kT_s\delta f)G_a + kT_n\delta f \quad (3.46a)$$

$$N_o = kG_a(T_s + T_n/G_a)\delta f = kG_a(T_s + T_e)\delta f \quad (3.46b)$$

$$T_e = T_n/G_a \quad (3.47)$$

Calculation

We consider the quadripole of Fig. 3.10, the input of which is connected to an available noise power generator kTsδf (Ts being the temperature of its internal resistance Rs and δf is an elementary band centered on the working frequency f).

In the ideal case (quadripole without noise, but having the same physical structure), we find at the output of quadripole the power (kTsδf) Ga; In fact, the quadripole adds noise and consequently the available power of output noise is No = (kTsδf) Ga + Nn, where Nn is the excess noise power added by quadripole.

The above relation can be put in the form of Eq. (3.46a), where Tn is the noise temperature corresponding to the power Nn. It should be noted that the gathering of noise powers is possible because the generator and quadripole noise are not correlated. Subtracting the factor (kδfGa), we obtain Eq. (3.46b); (3.47) is the definition of *the equivalent input noise temperature*.

The noise of a quadripole

- **Equivalent noise temperature at the input**

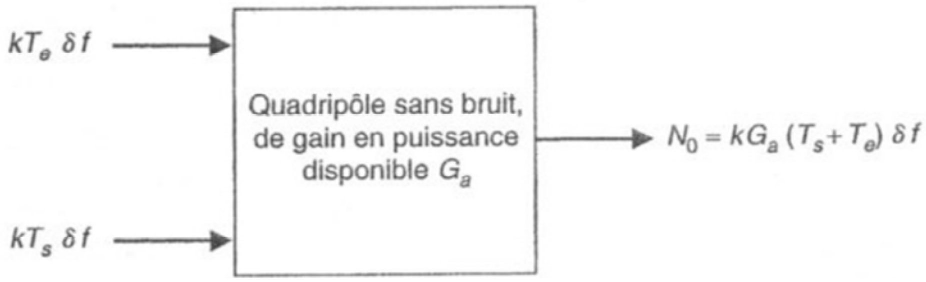


Fig.3.11

The expression (3.47) leads to the noise model in Fig. 3.11, where the noise power is put in the form of a sum of two noise powers applied to the input.

Consequences

1) The equivalent temperature of input noise does not depend on the source temperature of the source, but depends on the internal impedance of the source.
2) Noise temperature depends on the frequency
3) In Te's definition, the load is supposed to be ideal, without noise.
4) In the case of the nonlinear quadripole, we can have multiple input frequencies corresponding to a single output frequency and vice versa. In this case, for each pair of input-output frequencies, an equivalent temperature of input noise is defined.
5) This parameter provides an indication to compare two different quadruples: the one with the lowest input noise temperature adds less noise to the signal passing through it.
6) The advantage of the equivalent noise temperature characterization is that the noise temperatures are additive. For example, if the source has a noise temperature Ts and the amplifier is characterized by an equivalent noise temperature in the Tamp input, then the equivalent noise temperature of the ste assembly: Teq = Ts + Tamp.

Remark

Through this artifice, we separate the quadripole noise from its electrical circuit, in the same way as we did for a resistor (see Fig. 2.1), when we placed a noise generator in series or in parallel with the R resistance, supposedly ideal.

The noise of a quadripole

- **The chain quadripoles connection**

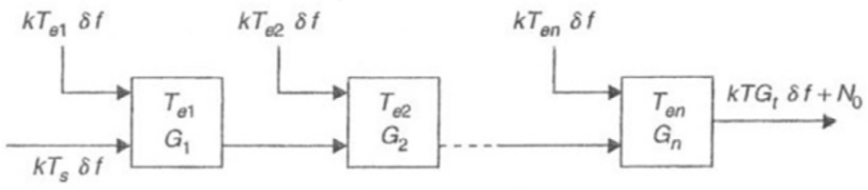


Fig.3.11

$$\left(\left(kT_{e1}G_1G_2\ldots G_n\right)+\left(kT_{e2}G_2\ldots G_n\right)+\left(kT_{en}G_n\right)\right)\delta f \quad (3.48)$$

$$G_t = G_1G_2\ldots G_n \quad (3.49)$$

$$T_{et} = T_{e1} + \frac{T_{e2}}{G_1} + \frac{T_{e3}}{G_1G_2} + \cdots + \frac{T_{en}}{G_1G_2\cdots G_{n-1}} \quad (3.50)$$

We consider more quadrupeds connected in chain, as in Fig. 3.12. , each being characterized by temperature Tei and the available gain Gi. We want to calculate the equivalent noise temperature at the input of the assembly, noted Tet.
The excess noise added by the quadripoles is Eq. (3.48).
The total gain in power is Eq. (3.49), specifying that the available gain of the first stage is measured with a generator having as internal resistance, the output resistance of the stage (i-1).

Using the same rationing as for Eq. (3.46b), the equivalent temperature of input noise of the chain is obtained by dividing the relation (3.48) by kGtδf. We get (3.50).
The result shows that, if the gain of the first stage is important (G1 >> 1), then the contributions of the next stages can be neglected (unless one of the stage is an attenuator).

The noise of a quadripole

- **Effective noise temperature**

$$\frac{N_{oL}}{G_t} = kT_{op}$$

$$T_{op} = \frac{N_{oL}}{kG_t} \quad (3.51)$$

$$\frac{S_o}{N_{0L}} = \frac{S_o/G_t}{N_{oL}/G_t} = \frac{S_i}{kT_{op}} \quad (3.51a)$$

$$T_{op} = T_e + T_s \quad (3.52)$$

The expression (3.46b) shows that the noise at the output of the quadripole depends on Te and Ts. In some cases, we can control Te, but in most cases, we have no control over the noise that reaches the quadripole input. For this reason, it seems interesting to have an unique parameter that characterizes the overall noise under real conditions, considering both Te and Ts temperatures at the same time. This unique parameter is the *effective noise temperature*, noted Top, and measured in Kelvin degrees.

Definition

According to IEEE, the noise performance of a given system can be evaluated according to the signal / noise ratio observed at its output. Let So be the output power of the signal in a unit band and Si the power available at the input of the quadripol. It is obvious that So = Si * Gt, where Gt is the transducing gain in power.

Similarly, the total output noise power output NoL, in a unit band, can be transposed to the input by dividing it by Gt; it serves to introduce the *effective noise temperature* with the relation (3.51).
In this case, the signal-to-noise ratio at the output is given by the relation (3.51a)

Consequences

1) For a linear quadripole adapted to the load, the effective noise temperature is Eq. (3.52).

2) The components of total NoL output power are:

- The signal source noise (modeled by Ts) that is transmitted at the output
- The noise generated in quadripole (modeled by Te) transmitted at the output
- The noise generated in the load, which is transmitted to quadripole and which is reflected back to the load due to missmatch.

The noise of a quadripole

- **The noise factor**

$$F = \frac{P_{ano}}{P_{an}} = \frac{P_{Rs} + P_Q}{P_{Rs}} = 1 + \frac{P_Q}{P_{Rs}} \quad (3.53)$$

Definition
Equivalent noise temperature Te is a useful parameter for characterizing thermal noise. At the same time, to estimate the signal-to-noise ratio at the output of a quadripole, we need to know both the noise delivered by the source and the available gain and bandwidth of the equivalent quadripole. Under these conditions, it is found that the noise temperature as a single parameter does not take into account the influence of cuadripole on the signals that cross it. To eliminate this inconvenience, the noise factor F is used.

The classic approach
The notion of noise factor is introduced naturally when all the noisy (ie resistive) elements of the quadripole are at the same temperature T0, which is generally ambient temperature.
Under these conditions, we will compare the real quadripole with a fictitious quadripole, with the identical physical structure, whose circuit elements (resistors, transistors, diodes, etc.) are assumed to be noise-free.
Applying always the same noise at the input, we can compare the available output noise power Pano, observed in reality, with Pan that would be available at the output of the ideal quadripole, without noise, if it was achievable (in the latter case, estimation of the noise at the output it is done by calculation, using the rules of circuit calculation). The noise factor is then defined with the relation (3.53), where:

P_Rs - the output noise power, generated by the resistance Rs of the signal source and amplified by the supposedly noisy quadripole
P_Q - power output noise, provided only by quadripole (supposedly closed at the iput on a noisy resistor Rs).

The noise of a quadripole

- **Properties of Noise factor**

1)The noise factor is independent of load resistance
2)The value of F depends on the resistance of the signal source
3)An ideal quadripole (no noise) has an F = 1
4)A real quadripole always adds its own noise to the one it receives from the source and this contribution is measured by quantity (F-1).

The noise of a quadripole

- **North's definition of the Noise Factor**

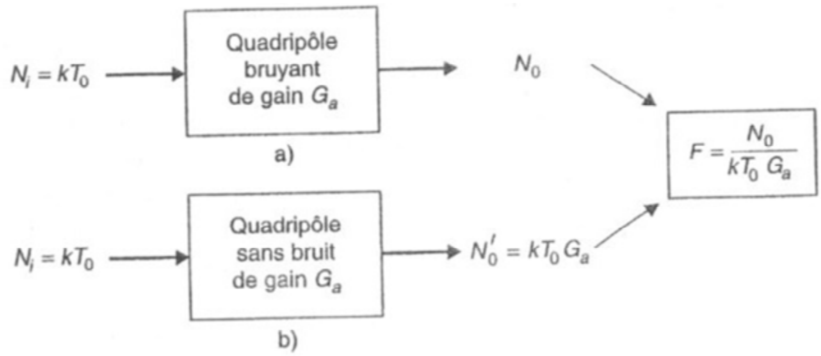


Fig.3.12

$$F = \frac{N_o}{N_o'} = \frac{N_o\left(\text{pentru T} = 290\text{ K}\right)}{kT_0 G_a} = 1 + \frac{N_n}{kT_0 G_a} > 1 \quad (3.54)$$

It is the definition adopted also by IEEE.
At a given frequency, the noise factor F of a quadripole is the ratio of the following two quantities:

1) Available output noise power No, in a unit band located at the working frequency, when the noise temperature of the dipole-generator connected to the input is kept constant and equal to the reference temperature T0 = 290 K, Fig.3.12a;
2) Part of (a) (denoted by No '), produced exclusively by the generator dipole connected to the input, at the working frequency, if the noise temperature of this dipole is maintained at T0 = 290 K and the quadripole is ideal, Fig. 3.12b.

We can express F in the form of the Eq, (3.54), where Nn is the noise power added by quadripole and Ga represents the available gain in power of the quadripole.

The noise of a quadripole

- **Friis definition for the noise factor**

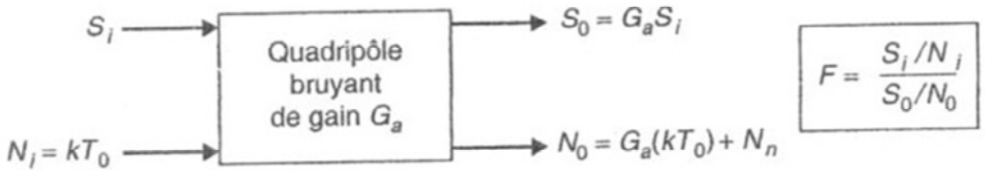


Fig.3.13

$$F = \frac{S_i/N_i}{S_o/N_o} = \frac{S_i/kT_0}{S_o/N_o} = \frac{1}{G_a}\frac{N_o}{kT_0} \quad (3.55)$$

Definition

The noise factor F of a quadripole is the ratio, at a specified frequency, between:

1) Signal / noise ratio available at the input terminals (when the equivalent noise temperature of the generator is T0 = 290 K and the band is limited by the quadripole itself)
2) The signal-to-noise ratio at the output of the quadripole.

This definition is illustrated in Fig. 3.13. In this definition, the noise factor is a measure of the deterioration of the signal / noise ratio caused by quadripole.
The definition of Friis implies a match at the input of the quadripole, which is not assumed at the exit.
Thus we have the relation (3.55), where Si, So are the available signal powers (in a unit band) at the input and output, and No and Ni = kT0 represent the available powers of noise at the output and input (the load is not considered). , in a unit band.

Remark

In the case of nonlinear quadripoles, where there are several output frequencies for one input frequency, we are required to consider a noise factor for each pair of frequencies. Moreover, the available power of output noise should not take into account the contributions of the image frequencies.

Comments

1) The notion of noise factor characterizes a quadripole only if it is accompanied by information regarding the internal impedance of the source that served in the measurements.
2) The noise factor can be expressed either as a dimensionless ratio or in decibels (dB).
3) The noise factor is defined at a reference noise temperature (which is usually equal to 290 K).

Limitations

1) If the internal impedance of the source is purely reactive, its noise is null and the resulting noise factor is infinite.
2) If the noise added by cuadripole is not significant compared to that of the source, the noise factor would appear that the ratio between two quantities is almost equal. In this case the calculation errors are unacceptable.
3) The noise factor depends on the frequency, polarization, temperature and resistance of the signal source. The comparison between two noise factors makes no sense if these conditions are not identical.

The noise of a quadripole

- **The relationship between F and Te**

$$F = \frac{1}{G_a}\frac{N_o}{kT_0\delta f} = \frac{N_n + kT_0 G_a \delta f}{G_a kT_0 \delta f} = 1 + \frac{N_o / G_a}{kT_0 \delta f} \quad (3.56)$$

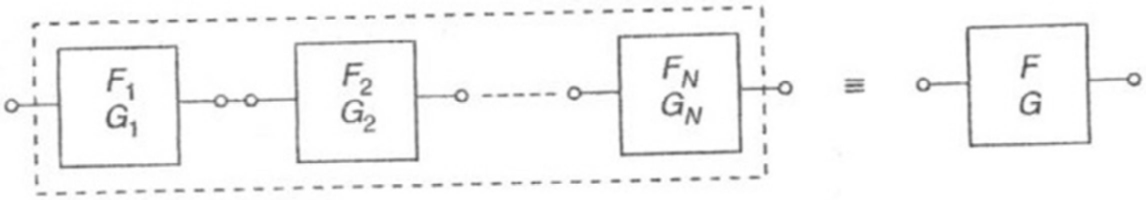
$$F = 1 + \frac{kT_n \delta f / G_a}{kT_0 \delta f} \quad (3.56a)$$

$$F = 1 + \frac{T_e}{T_0} \quad (3.57) \qquad T_e = T_0 (F - 1) \quad (3.58)$$

According to the IEEE definition, for an elementary band δf one can write Eq. (3.56).
After Eq. (3.46a) , we have Nn = kTnδf, whence Eq. (3.56a)
In this case, the noise factor is put in the form of Eq. (3.57), where we remember that Te is the equivalent temperature of noise at the input. In an equivalent form, this relation can be written as Eq.(3.58).

The noise of a quadripole

- **Chain of quadripols**



$$F = F_1 + \frac{F_2 - 1}{G_1} + \frac{F_3 - 1}{G_1 G_2} + \ldots + \frac{F_N - 1}{G_1 G_2 \ldots G_{N-1}} \quad (3.59)$$

In the case of N stages connected in the chain (Fig. 3.14), for which the individual noise factor is known, it is useful to calculate the global noise factor.
This problem is of great practical importance, because the noise of the input stage of an amplifier is of great importance for the noise from the output.
If we replace in Eq. (3.50) noise temperatures by its expression (3.58), we obtain Friis's formula (3.59).

Eq. (3.59) is valid under the following conditions:
1) The noise factors Fi should be evaluated considering the output impedance of the floor (i-1) as the internal impedance of the generator supplying the first floor.
2) All quadripoles must be linear.
3) Each quadripole must have the real part of the positive output impedance
4) Each quadripole should add noise to the system.

The noise of a quadripole

- **Effective noise factor**

$$F_{op} = \frac{N_o}{G\left(kT_0\delta f\right)} = \frac{T_{op}}{T_0} \quad (3.60)$$

The expression (3.46b) shows that the noise at the output of the quadripole depends on Te and Tn. In some cases we can control Tn, but in most cases we cannot control the noise that reaches the quadripole input. In this case, it is useful to have a unique parameter that characterizes the overall noise under real conditions, considering both Te and Tn temperatures simultaneously. This parameter is the effective noise temperature, noted with Top, measured in Kelvin.

Definition

The effective noise factor Fop is defined as the ratio between the following quantities:

1) The available noise output power, in an elementary frequency band δf, at normal working temperatures
2) The available power output noise, due only to the input generator considered at reference temperatures.

Thus we have Eq. (3.60), where Top is the effective quadrupole noise temperature and T0 = 290 K.

The only difference with the definition of the classic noise factor (North definition) is that at that time it is no longer required for the quadripole to be at the reference temperature T0.

The noise of a quadripole

- **Noise factor: Extended definition**

$$F_e = \frac{N_{en}}{G_e\left(kT_0\delta f\right)} \quad (3.61)$$

The definition of the noise factor given by North and then adapted by IEEE, does not raise any problem as long as the resistance of the generating dipole is the output resistance of the passive quadripole. At the same time, in many of the practical situations, one may encounter the situation where the output resistance is negative, as in the case of the tunnel diode amplifier.
Regarding the resistance of the generator, priori it is always positive, but it is sufficient to consider the case of a cascade of several quadruples, where the output resistance of the first stage becomes the resistance of the generator for the stage (i + 1), to realize that this assumption is not an absolute rule. Then, it seems logical to consider the possibility of having negative values for both the output resistance and that of the generator.

If we have negative values for these resistors, we can no longer use, in the definition of the noise factor, the notion of available power. As discussed in a previous slide in this chapter, it is obvious that we must replace the available powers with the exchangeable powers. The exchangeable power offers the advantage of thet they identifying with the available power in the case of positive resistors, but in the case of negative ones, it becomes negative, its extreme value retaining the same absolute value, finite.

Definition

The extended noise factor, denoted by Fe, is introduced by the relation (3.61), which is similar to the classical definition, except that now Nen represents the exchangeable power of output noise, with the input generator maintained at the reference temperature $T0 = 290$ K and Ge is the gain in exchangeable power.

The noise of a quadripole

- **Noise measure**

$$M = \frac{F - 1}{1 - 1/G} \quad (3.62)$$

Definition of Van der Ziel

The noise measure, denoted by M, of a quadripole having the noise factor F and the gain available in power G, is introduced by Eq. (3.62).