ZGOMOTE SI PERTURBATII

Cap.1
Notiuni fundamentale

Studiul fluctuatiilor folosind teoria de semnal

Semnale

- Semnale periodice

$$v(t) = v(t+T) \qquad (1.1)$$

- Semnale aperiodice

Valoarea medie

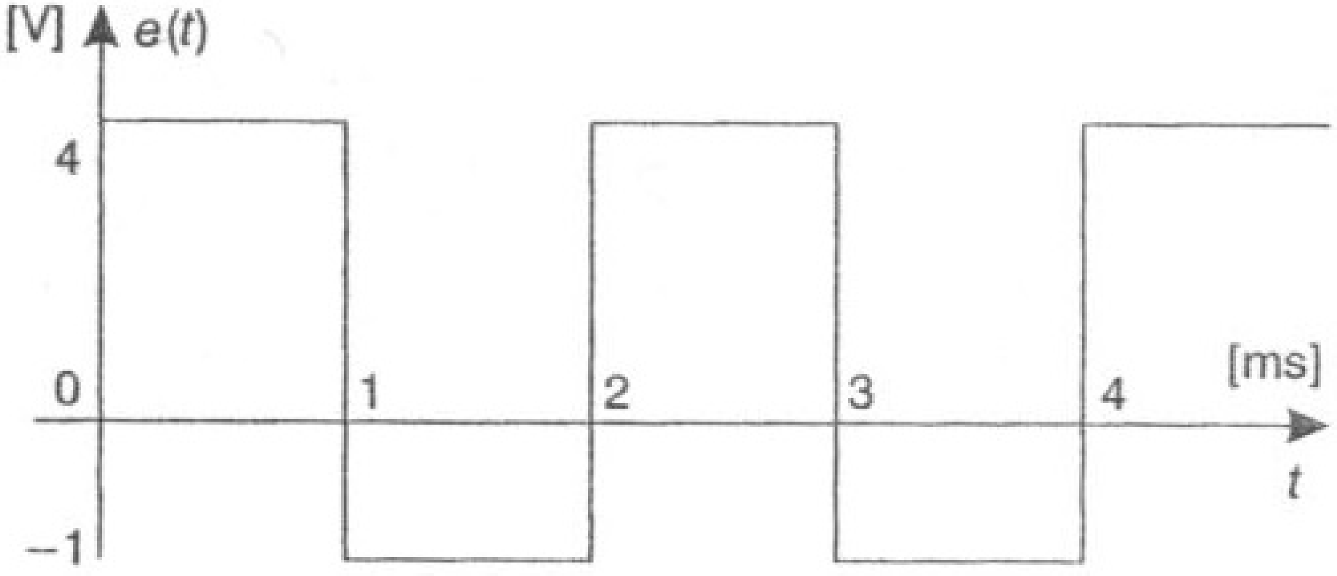


Fig.1.1

$$I_0 = \frac{1}{T}\int_0^T i(t)\,dt \qquad (1.2)$$

Exemplu

$$E_0 = \frac{1}{(2ms)}\left(\left(4V\right)\left(1ms\right)+\left(-1V\right)\left(1ms\right)\right)=1.5V$$

Valoarea medie patratica

Valoarea eficace

$$\underbrace{\frac{1}{R}\frac{1}{T}V^2\int_0^T \cos^2 \omega t dt}_{\text{Puterea semnalului}} = \underbrace{\frac{1}{R}V_{CC}^2}_{\text{Puterea in CC}} \qquad (1.3)$$

$$V_{ef} = V_{CC} = V/\sqrt{2} \qquad (1.4)$$

Valoarea medie patratica

$$V_{qm} = \left(V_{ef}\right)^2 = \frac{1}{T}\int_0^T v^2(t)\,dt \qquad (1.5)$$

Calculul valorii medii patratice

- Ridicati la patrat semnalul original
- Calculati aria cuprinsa sub aceasta curba pentru o perioada
- Impartiti aria la perioada

Exemplu

$$E_{qm} = \frac{\left(4V\right)^2\left(1ms\right) + \left(-1V\right)^2\left(1ms\right)}{\left(2ms\right)} = 8.5\,V^2$$

$$E_{ef} = \sqrt{8.5} = 2.91\,V$$

$$P = E_{qm}/R \qquad (1.6)$$

Corelatia

$$v_a = A\sin(\omega t) \qquad (1.7a)$$

$$v_b = B\sin(\omega t + \phi) \quad (1.7b)$$

$$v_b = \underbrace{(B\cos\phi)\sin(\omega t)}_{\text{termenul 1}} + \underbrace{(B\sin\phi)\cos(\omega t)}_{\text{termenul 2}} \quad (1.7c)$$

Cazul fluctuatiilor

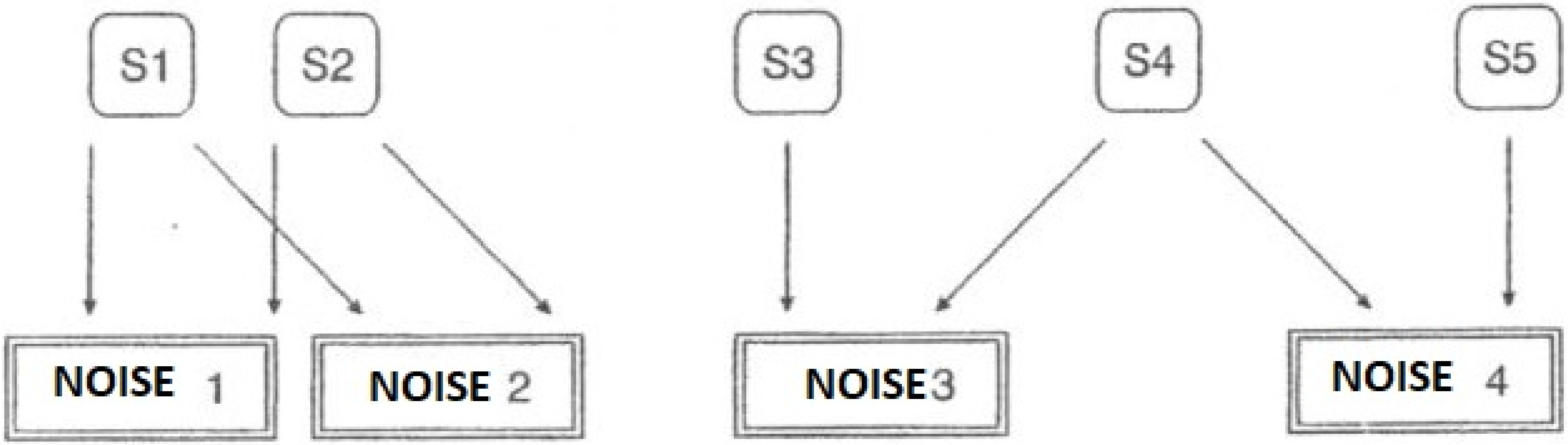


Fig.1.2

Coeficientul de corelatie (aproximarea clasica)

$$\overline{v^2} = \overline{\left(v_a + v_b\right)^2} = \overline{v_a^2} + 2\overline{v_a v_b} + \overline{v_b^2} \qquad (1.8a)$$

$$\overline{v^2} = \frac{1}{T}\int_0^T \left(A^2 \sin^2 \omega t + 2AB \sin(\omega t)\sin(\omega t + \phi) + B^2 \sin^2(\omega t + \phi)\right) dt \qquad (1.8b)$$

$$\overline{v^2} = \frac{A^2}{2} + AB\cos\phi + \frac{B^2}{2} \qquad (1.8c)$$

Coeficientul de corelatie (aproximarea clasica)

$$\overline{v^2} = v_{ef}^2 = \left(\frac{A}{\sqrt{2}} + \frac{B}{\sqrt{2}}\right)^2 \qquad (1.8d)$$

$$\overline{v^2} = \frac{A^2}{2} + \frac{B^2}{2} \qquad (1.8e)$$

$$c = \frac{\overline{v_a v_b}}{\sqrt{\overline{v_a^2}\,\overline{v_b^2}}} \qquad (1.9)$$

$$c = \frac{\overline{v_a v_b}}{(v_a)_{ef}(v_b)_{ef}} = \frac{AB\cos\phi}{AB} = \cos\phi \qquad (1.10)$$

Studiul fluctuatiilor folosind teoria probabilitatilor

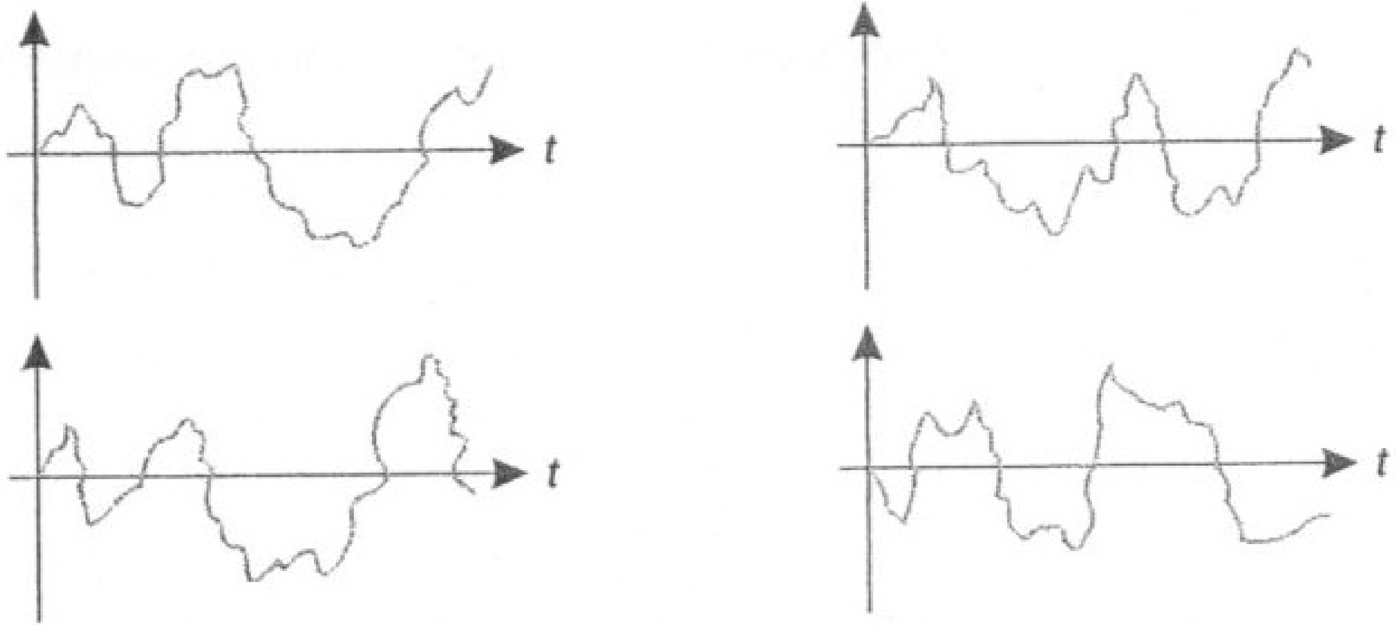


Fig.1.3

Studiul fluctuatiilor folosind teoria probabilitatilor

- **Densitate de probabilitate**

$$f(x) = \lim_{\substack{\Delta x \to 0 \\ N \to \infty}} \frac{(\text{Numarul de valori in intervalul } \Delta\text{x situat la x}) \cdot \Delta x}{\text{Numarul total N de valori}}$$

- **Fuctia de repartitie**

$$F(x) = \int_{-\infty}^{x} f(y)\,dy \qquad (1.11)$$

Caracterizarea marimilor fluctuante

- Ergodism
- Stationaritate
- Medii

$$m_n(x) = \overline{x^n(t)} = \int_{-\infty}^{+\infty} x^n f(x)\,dx \quad (1.12)$$

$$m_2(x-\overline{x}) = \text{var.}\{x\} = \overline{(x-\overline{x})^2} = \overline{x^2} - (\overline{x})^2 \quad (1.13)$$

$$\sigma_x = \sqrt{\text{var.}(x)} \quad (1.14)$$

Sensul fizic asociat diverselor val. medii

1. Media de ordinal intii reprezinta component de current continuu.
2. Matratul mediei de ordinal intii poate fi identificat cu puterea de current continuu dezvoltata intr-o rezistenta de 1Ω.
3. Media de ordinal doi reprezinta puterea totala disipata intr-o rezistenta de 1Ω.
4. Varianta este puterea componentei de semnal disipata in rezistenta de 1Ω.
5. Deviatia standard reprezinta valoarea efectiva a componentei de semnal a curentului.

Teorema limitei centrale

$$Y = \sum_{i}^{n} X_i$$

$$Y \xrightarrow{n \to \infty} \text{Variabila normala}$$

$$\langle Y \rangle = n \langle X_i \rangle \text{, var.}(Y) = n\sigma^2$$

Variabila normala $$f(x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left\{ -\frac{\left(x - \bar{x}\right)^2}{2\sigma^2} \right\}$$

Caracterizarea folosind doua variabile

- Densitatea de probabilitate f(x,y)
- Functia de repartitie F(x,y)

$$F\left(x, y\right) = \int_{-\infty}^{x} \int_{-\infty}^{y} f(u,v)dudv \quad \left(1.15\right)$$

Caracterizarea folosind doua variabile

- Medii

$$m_{ik}(x,y) = \overline{x^i y^k} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} x^i y^k f(x,y)dxdy \quad (1.16)$$

$$m_{00}(x,y) = 1 \quad (1.17a)$$

$$m_{01} = \overline{y} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} yf(x,y)dxdy \quad (1.17b)$$

$$m_{10} = \overline{x} = \int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} xf(x,y)dxdy \quad (1.17c)$$

Caracterizarea folosind doua variabile

- Variabile centrate

$$\mu_{ik}(x,y)=\overline{\left(x-\overline{x}\right)^{i}\left(y-\overline{y}\right)^{k}}=\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty}\left(x-\overline{x}\right)^{i}\left(y-\overline{y}\right)^{k}f(x,y)dxdy \quad (1.18)$$

$$\mu_{02}=\overline{\left(y-\overline{y}\right)^{2}}$$

$$\mu_{20}=\overline{\left(x-\overline{x}\right)^{2}}$$

Covarianta

$$\mu_{11} = \overline{\left(x-\overline{x}\right)\left(y-\overline{y}\right)} = \overline{xy} - \overline{x}\,\overline{y} \quad (1.19)$$

Corelatia

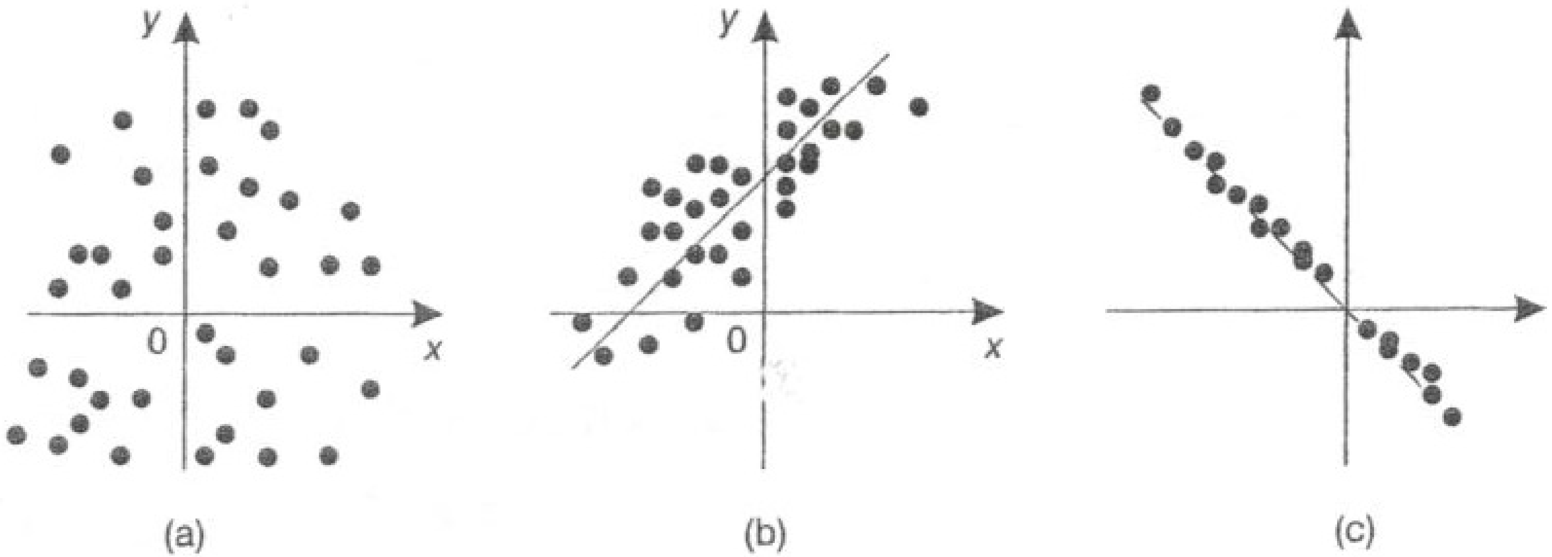


Fig.1.4

$$c = \frac{\mu_{11}}{\sigma_x \sigma_y} \quad (1.20) \qquad -1 \le c \le +1$$

Corelatia partiala

$$y = ax + z, \quad a = \text{ct.} \quad (1.21a)$$

unde

$$\overline{x} = \overline{y} = \overline{z} \quad (1.21b)$$

$$c = (sign\ a)\left(1 + \overline{x^2}/\overline{a^2x^2}\right)^{-1/2} \quad (1.22)$$

Functia de autocorelatie

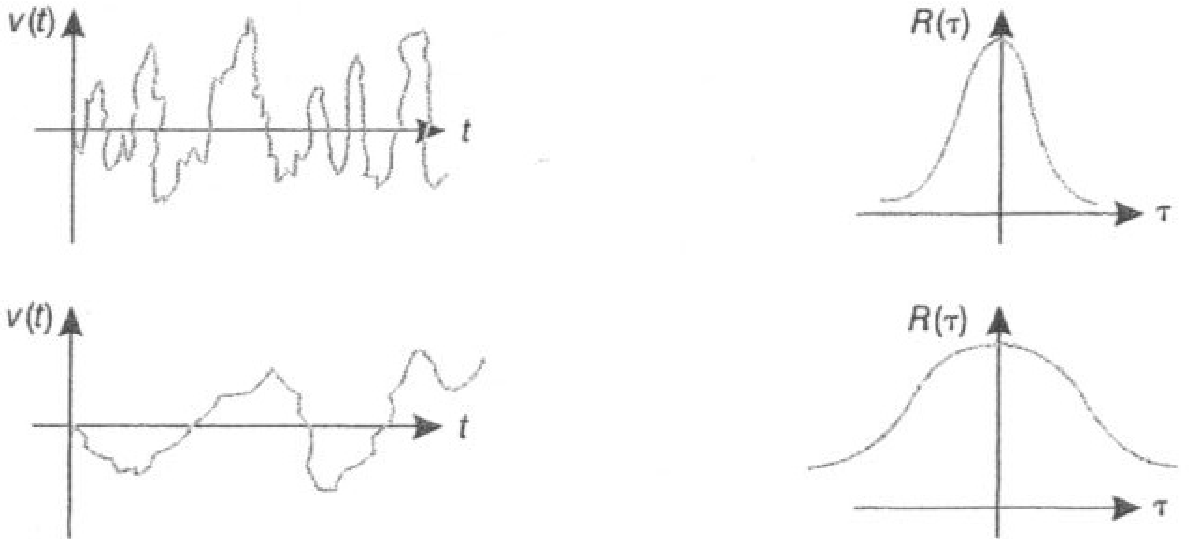


Fig.1.5

$$R(\tau)=\overline{v(t_1)v(t_1+\tau)} \quad (1.23)$$

$$R(0)=\overline{v^2} \quad (1.24)$$

Spectre de energie si de putere

- **Transformata Fourier**

$$F(f)=\int_{-\infty}^{+\infty} f(t)\exp(-j2\pi ft)dt \quad (1.25a)$$

$$f(t)=\int_{-\infty}^{+\infty} F(f)\exp(j2\pi ft)df \quad (1.25b)$$

Spectrul unui semnal aperiodic

$$S_f(V) = \frac{V_{ef}^2}{\Delta f} = \left(\frac{V_{ef}}{\sqrt{\Delta f}} \right)^2 \quad (1.26)$$

- Densitate spectrala
- Putere normalizata

Teorema lui Parseval

$$\int_{-\infty}^{+\infty} x_1(t)\, x_2(t)\, dt = \int_{-\infty}^{+\infty} X_1^*(f)\, X_2(f)\, df \quad (1.27)$$

$$W = \int_{-\infty}^{+\infty} \overline{v^2(t)}\, dt = \frac{1}{2\pi} \int_{-\infty}^{+\infty} F(j\omega)\, F^*(j\omega)\, d\omega = \frac{1}{2\pi} \int_{-\infty}^{+\infty} \left| F(j\omega) \right|^2 d\omega \quad (1.28)$$

Densitate spectrala de energie

$$S_{\omega}(W) = \frac{1}{2\pi}\left|F(j\omega)\right|^2 \qquad (1.29)$$

Analiza armonica a marimilor fluctuante

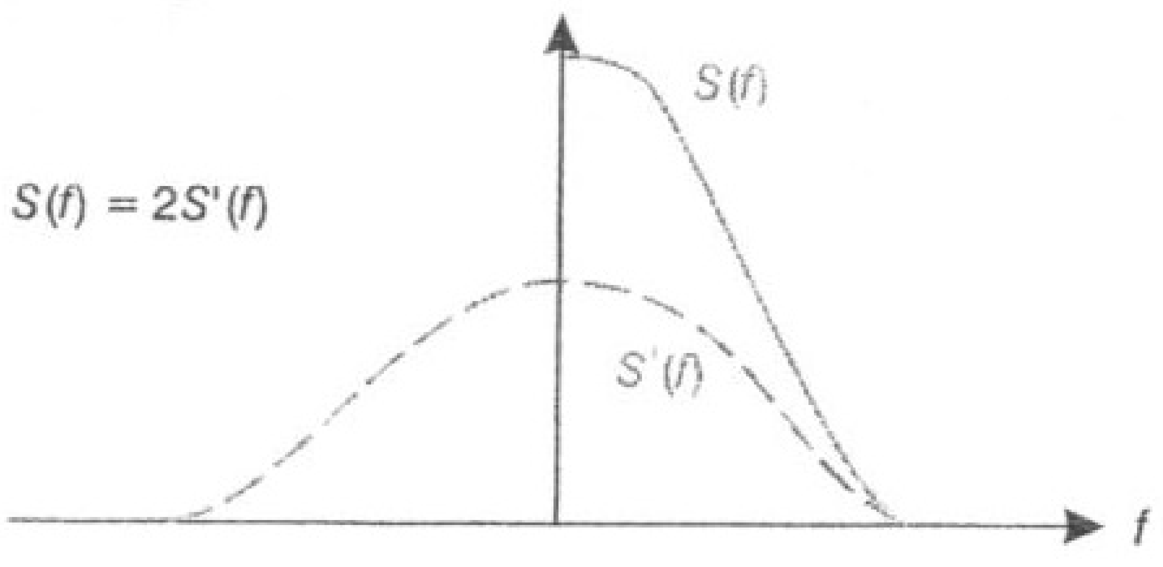


Fig.1.6

Teorema Wiener - Khintchine

$$S_f(x) = \int_{-\infty}^{+\infty} R(\tau)\exp(-j\omega\tau)\,d\tau \quad (1.30)$$

$$R(\tau) = \int_{-\infty}^{+\infty} S_f \exp(+j\omega\tau)\,df \quad (1.31)$$

Superpozitia mai multor fluctuatii

$$z(t) = x(t) + y(t)$$

$$\overline{z^2(t)} = \overline{\left(x(t) + y(t)\right)^2} = \overline{x^2(t)} + 2\overline{x(t)\,y(t)} + \overline{y^2(t)}$$

$$\overline{z^2(t)} = P_1 + 2P_{12} + P_2 \quad (1.32)$$

Functia de intercorelatie
Densitate spectrala de putere incrucisata

$$S_f(xy) = F\{R_{xy}(\tau)\} \quad (1.33)$$

Cazul sistemelor liniare

$$S_f(y) = H(j2\pi f)H^*(j2\pi f)S_f(x) = \left|H(j2\pi f)\right|^2 S_f(x) \quad (1.34)$$

$$R_y(\tau) = \mathcal{F}^{-1}\{S_f(y)\} = \int_{-\infty}^{+\infty} \left|H(j2\pi f)\right|^2 S_f(x)\exp(j2\pi/\tau)df \quad (1.35)$$

$$R_{xy}(\tau) = h(\tau) \circ R_x(\tau) \quad (1.36)$$

$$S_f(xy) = H(j2\pi f)S_f(x) \quad (1.37)$$

$$S_f(yx) = S_f^*(xy) \quad (1.38)$$

Concluzie

Exceptind fenomenele tranzitorii, teoria clasica a circuitelor ramine valabila si pentru fluctuatii. Atit timp cit scopul cautat Este calculul valorilor patratice medii (ale tensiunii sau curentului) si nu calculul tensiunilor (curentilor) circuitului.

Matricea de corelatie

- **Spectru de puteri proprii si incrucisat**

$$z(t)=x(t)+y(t)$$

$$|Z|^2=(X+Y)\left(X^*+Y^*\right)=|X|^2+|Y|^2+XY^*+X^*Y \quad (1.39)$$

$$S'_f(Z)=\lim_{\tau\to\infty}\frac{|Z|^2}{\tau}=S'_f(XX)+S'_f(YY)+S'_f(XY)+S'_f(YX) \quad (1.40)$$

$$S'_f(XX)=\lim_{\tau\to\infty}\frac{X^*X}{\tau} \quad \text{si} \quad S'_f(YY)=\lim_{\tau\to\infty}\frac{Y^*Y}{\tau} \quad (1.41)$$

$$S'_f(XY)=\lim_{\tau\to\infty}\frac{X^*Y}{\tau} \quad \text{si} \quad S'_f(YX)=\lim_{\tau\to\infty}\frac{Y^*X}{\tau} \quad (1.42)$$

Matricea de corelatie

- **Propietati**

$$S'_f(Z) = S'_f(XX) + S'_f(YY) + 2\Re\left[S'_f(XY)\right] \quad (1.43a)$$

$$S_f(Z) = S_f(XX) + S_f(YY) + 2\Re\left[S_f(XY)\right] \quad (1.43b)$$

Teorema Wiener – Khintchine 2

$$S_f(XX) = 2\int_{-\infty}^{+\infty} \overline{x(t)\,x(t+s)} \exp(j\omega s)\,ds \quad (1.44a)$$

$$S_f(YY) = 2\int_{-\infty}^{+\infty} \overline{y(t)\,y(t+s)} \exp(j\omega s)\,ds \quad (1.44b)$$

$$S_f(XY) = 2\int_{-\infty}^{+\infty} \overline{x(t)\,y(t+s)} \exp(j\omega s)\,ds \quad (1.44c)$$

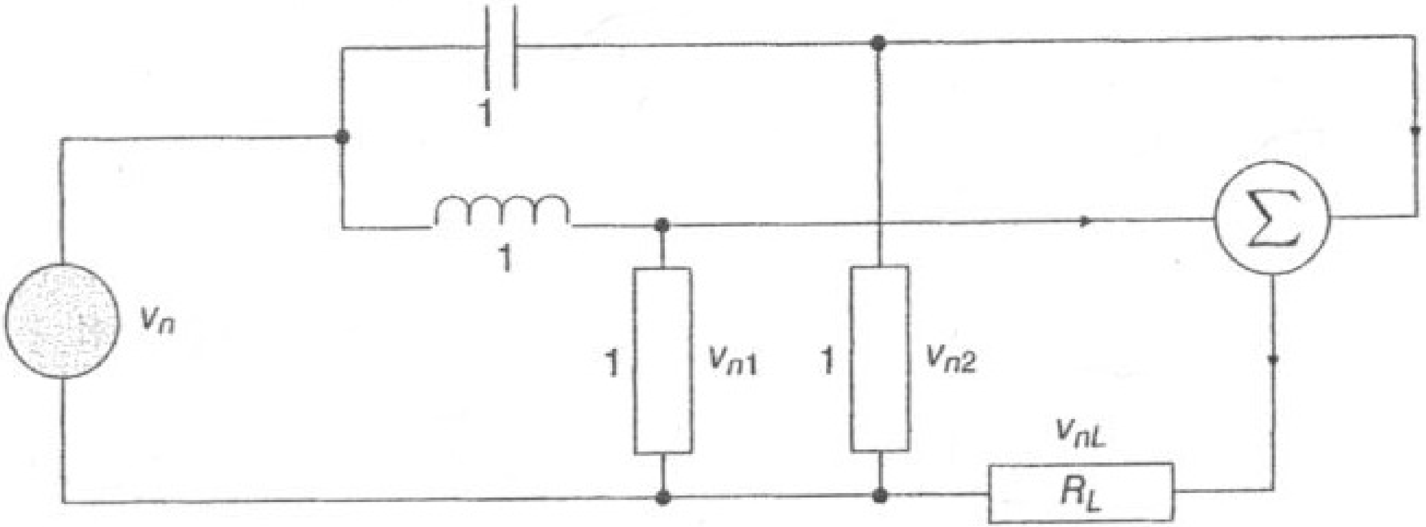
Matricea de corelatia

$$C = \begin{bmatrix} C_{11} & C_{12} \\ C_{21} & C_{22} \end{bmatrix} = \begin{bmatrix} S_f(XX) & S_f(XY) \\ S_f(YX) & S_f(YY) \end{bmatrix} \quad (1.45)$$

Coeficientul de corelatie (aproximatia Kleckner)

$$\Gamma_{\omega}(i, j) = \frac{S_{\omega}(i, j)}{\sqrt{S_{\omega}(i,i)\, S_{\omega}(j, j)}} \qquad (1.46)$$

Exemplu



$$2)\ \Gamma_\omega(1,2) = -j$$

Fig.1.7

$$v_{n1} = \frac{1}{1+j\omega} v_n \quad \text{deci} \quad |v_{n1}|^2 = \frac{1}{1+\omega^2} v_n^2$$

$$v_{n2} = \frac{1}{1+1/j\omega} v_n \quad \text{deci} \quad |v_{n2}|^2 = \frac{\omega^2}{1+\omega^2} v_n^2$$

$$v_{n1} = \frac{1}{1+j\omega} v_n \quad \text{deci} \quad |v_{n1}|^2 = \frac{1}{1+\omega^2} v_n^2$$

$$v_{n2} = \frac{1}{1+1/j\omega} v_n \quad \text{deci} \quad |v_{n2}|^2 = \frac{\omega^2}{1+\omega^2} v_n^2$$

$$S_\omega(1,1) = \overline{v_{n1} v_{n1}^*} = \overline{|v_{n1}|^2} = \frac{\overline{v_n^2}}{1+\omega^2} = \frac{K}{1+\omega^2}$$

$$S_\omega(2,2) = \overline{v_{n2} v_{n2}^*} = \overline{|v_{n2}|^2} = \frac{\omega^2}{1+\omega^2} \overline{v_n^2} = \frac{\omega^2}{1+\omega^2} K$$

$$S_\omega(1,2) = \overline{v_{n1} v_{n2}^*} = \overline{|v_{n2}|^2} = \overline{\frac{1}{1+j\omega} v_n \frac{1}{1-1/j\omega} v_n} = \frac{-j\omega}{1+\omega^2} K$$