

CUPLOARE

Proprietati de baza ale cuploarelor directionale

Circuite cu patru porti

$$\begin{array}{ll} (S_{ij} = S_{ji}) & \text{Reciproc} \\ S_{ii} = 0 & \text{Adaptare simultana la toate portile} \end{array} \quad > \quad [S] = \begin{bmatrix} 0 & S_{12} & S_{13} & S_{14} \\ S_{12} & 0 & S_{23} & S_{24} \\ S_{13} & S_{23} & 0 & S_{34} \\ S_{14} & S_{24} & S_{34} & 0 \end{bmatrix}$$

$$(11) \quad S_{14}^* \left(|S_{13}|^2 - |S_{24}|^2 \right) = 0$$

$$(13) \quad S_{23} \left(|S_{12}|^2 - |S_{34}|^2 \right) = 0$$

$$(14a) \quad |S_{12}|^2 + |S_{13}|^2 = 1$$

$$(14b) \quad |S_{12}|^2 + |S_{24}|^2 = 1$$

$$(14c) \quad |S_{13}|^2 + |S_{34}|^2 = 1$$

$$(14d) \quad |S_{24}|^2 + |S_{34}|^2 = 1$$

$$(15) \quad S_{12}^* S_{13} + S_{24}^* S_{34} = 0$$

+

$$\sum_k S_{ik} S_{kj}^* = \delta_{ij} \quad \text{Fara pierderi}$$

||

< **10 ecuatii**

Cazul 1

$$(11) \text{ si } (13) > S_{14} = S_{23} = 0 \Rightarrow [S] = \begin{bmatrix} 0 & S_{12} & S_{13} & 0 \\ S_{12} & 0 & 0 & S_{24} \\ S_{13} & 0 & 0 & S_{34} \\ 0 & S_{24} & S_{34} & 0 \end{bmatrix} \Leftrightarrow \text{Cuplor directional}$$

$$(14a) \text{ si } (14b) > |S_{13}| = |S_{24}| \quad \text{Alegem: } S_{12} = S_{34} = \alpha \quad S_{13} = \beta e^{j\theta} \quad S_{24} = \beta e^{j\phi}$$

$$(14b) \text{ si } (14d) > |S_{12}| = |S_{34}|$$

$$(15) > \theta + \phi = \pi \pm 2n\pi$$

$$\text{Cuplor simetric} \quad \theta = \phi = \pi/2$$

$$\text{Cuplor antisimetric} \quad \theta = 0, \phi = \pi$$

$$[S] = \begin{bmatrix} 0 & \alpha & j\beta & 0 \\ \alpha & 0 & 0 & j\beta \\ j\beta & 0 & 0 & \alpha \\ 0 & j\beta & \alpha & 0 \end{bmatrix}$$

$$[S] = \begin{bmatrix} 0 & \alpha & \beta & 0 \\ \alpha & 0 & 0 & -\beta \\ \beta & 0 & 0 & \alpha \\ 0 & -\beta & \alpha & 0 \end{bmatrix}$$

$$(14a) > \alpha^2 + \beta^2 = 1$$

Cazul 2

$$(11) \text{ si } (13) \Rightarrow \begin{cases} |S_{13}| = |S_{24}| \\ |S_{12}| = |S_{34}| \end{cases} \quad \text{Alegem: } S_{13} = S_{24} = \alpha \quad S_{12} = S_{34} = j\beta$$
$$(14a) \Rightarrow \alpha^2 + \beta^2 = 1$$

$$S_{13}^* S_{23} + S_{14}^* S_{24} = 0 \Rightarrow \alpha(S_{23} + S_{14}^*) = 0$$

$$\longrightarrow S_{14} = S_{23} = 0 \quad \text{Cuplor directional}$$

$$S_{12}^* S_{23} + S_{14}^* S_{34} = 0 \Rightarrow \beta(S_{14}^* - S_{23}) = 0$$



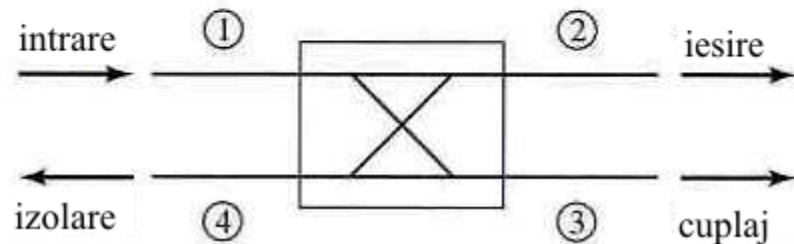
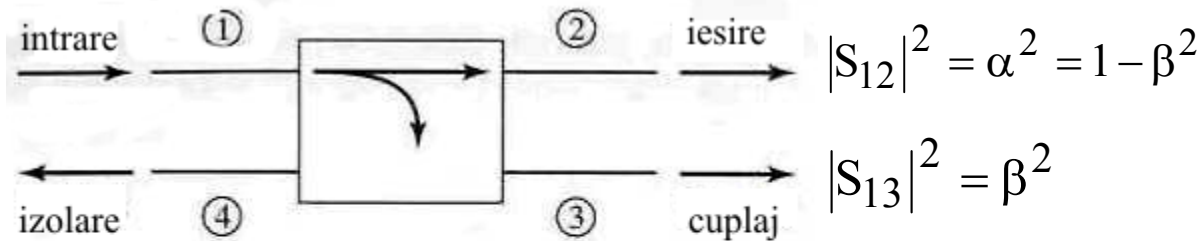
$$\alpha = \beta = 0 \quad \text{Caz banal}$$

$$[S] = \begin{bmatrix} 0 & j\beta & \alpha & 0 \\ j\beta & 0 & 0 & \alpha \\ \alpha & 0 & 0 & j\beta \\ 0 & \alpha & j\beta & 0 \end{bmatrix}$$

CONCLUZIE

**Orice circuit cu patru porti,
reciproc, fara pierderi si adaptat la toate portile
este un cuplor directional**

Cuplor directional



$$\text{Cuplaj} = C = 10 \log \frac{P_1}{P_3} = -20 \log(\beta) \text{ dB}$$

$$\text{Directivitate} = D = 10 \log \frac{P_3}{P_4} = 20 \log \left(\frac{\beta}{|S_{14}|} \right) \text{ dB}$$

$$\text{Izolare} = I = 10 \log \left(\frac{P_1}{P_4} \right) = -20 \log |S_{14}| \text{ dB}$$

$$I = D + C, \text{ dB}$$

Cuplor hibrid

Cuplorul hibrid este cuplorul direccional de 3 dB

$$\alpha = \beta = 1/\sqrt{2}$$

Cuplor hibrid in cuadratura

$$(\theta = \phi = \pi/2)$$

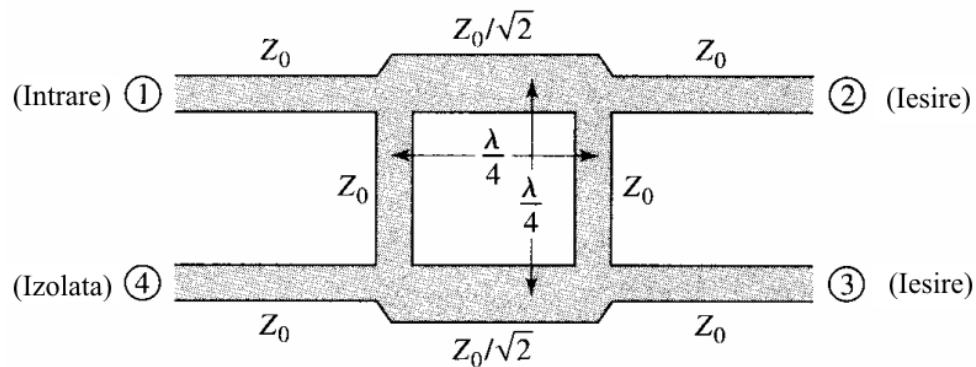
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

Cuplor hibrid in inel

$$(\theta = 0, \phi = \pi)$$

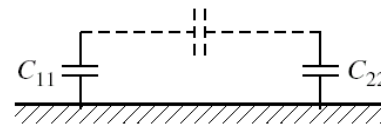
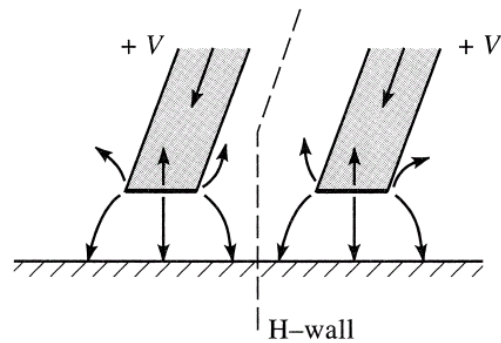
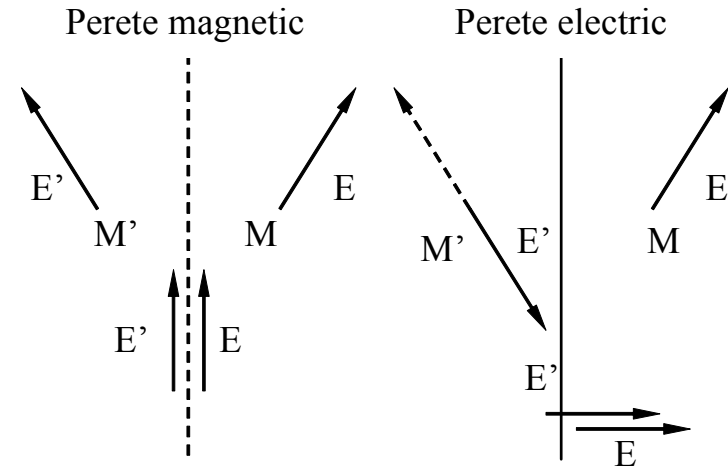
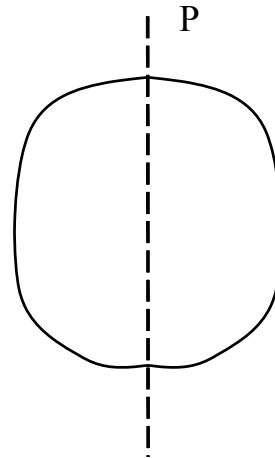
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & 1 & 0 \\ 1 & 0 & 0 & -1 \\ 1 & 0 & 0 & 1 \\ 0 & -1 & 1 & 0 \end{bmatrix}$$

Cuplorul hibrid în cuadratură (90°)

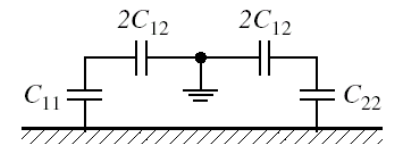
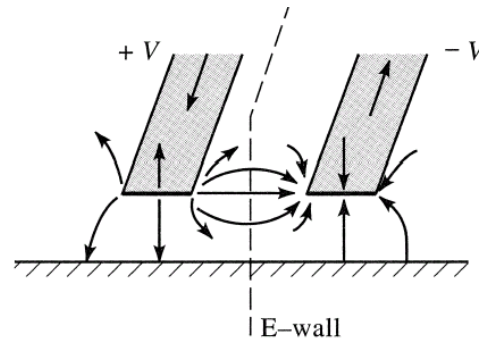


$$[S] = \frac{-1}{\sqrt{2}} \begin{bmatrix} 0 & j & 1 & 0 \\ j & 0 & 0 & 1 \\ 1 & 0 & 0 & j \\ 0 & 1 & j & 0 \end{bmatrix}$$

Simetria fizică se transformă în simetrie sau antisimetrie a câmpului electromagnetic

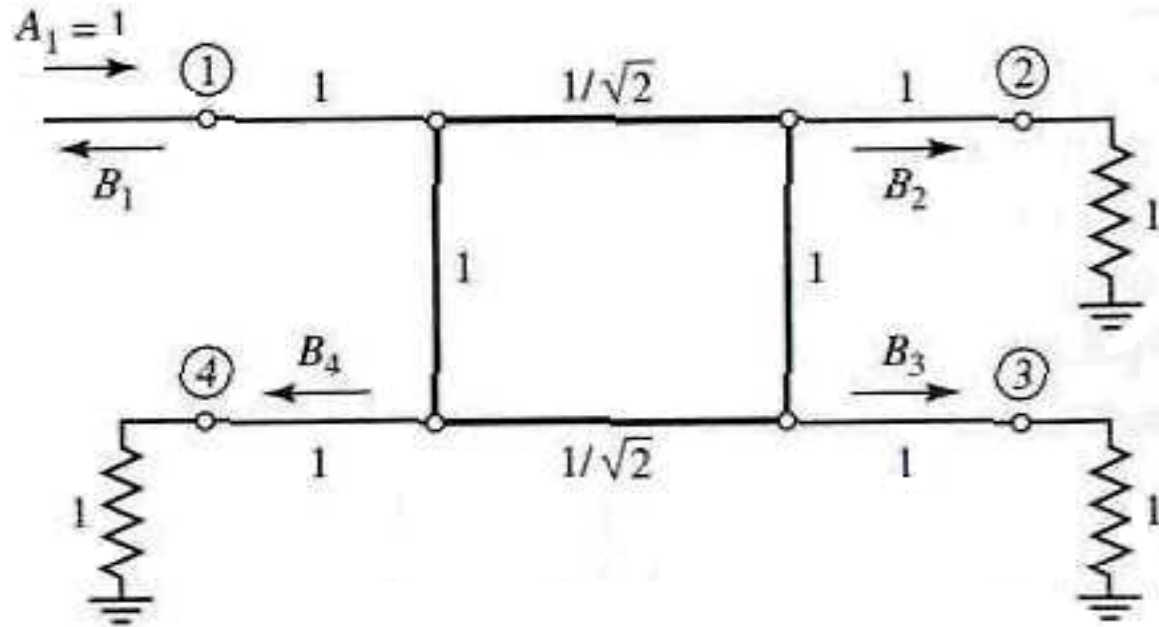


(a)

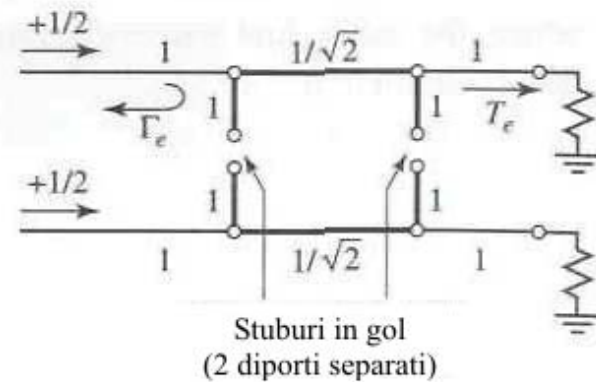
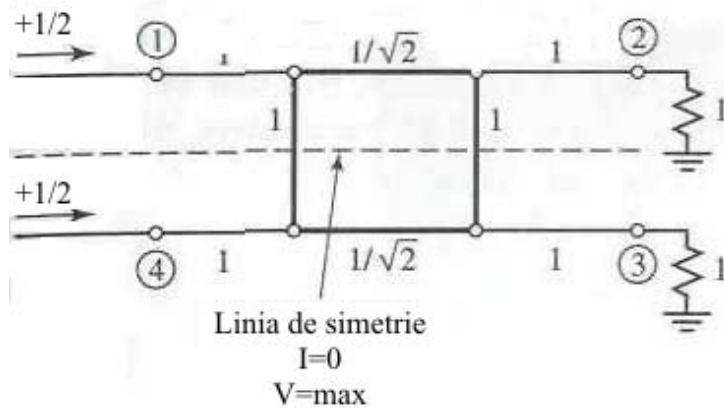


(b)

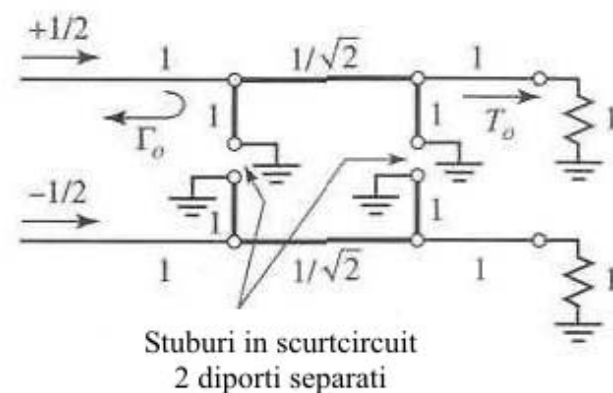
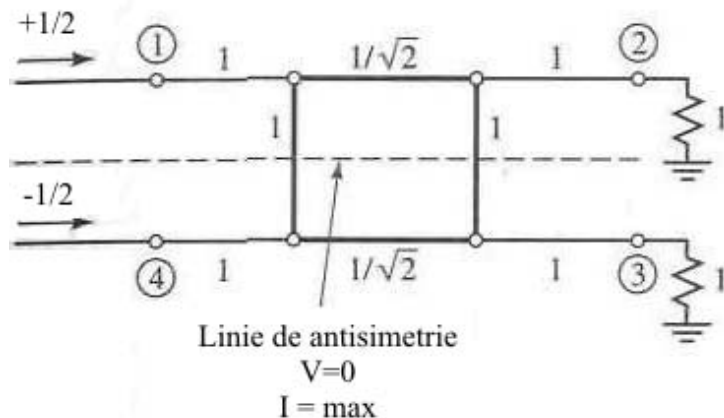
Analiza pe modul par-impar



Analiza pe modul par-impar



(a)



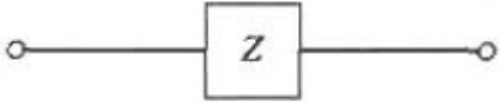
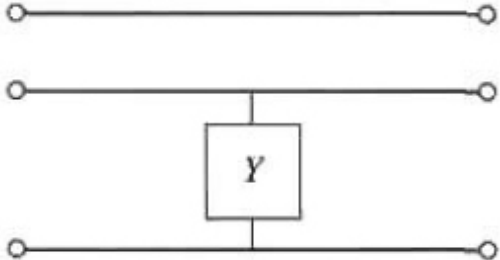
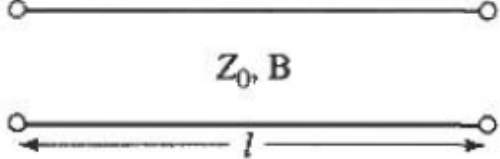
(b)

$$b_1 = \frac{1}{2}\Gamma_e + \frac{1}{2}\Gamma_o$$

$$b_2 = \frac{1}{2}T_e + \frac{1}{2}T_o$$

$$b_3 = \frac{1}{2}T_e - \frac{1}{2}T_o$$

$$b_4 = \frac{1}{2}\Gamma_e - \frac{1}{2}\Gamma_o$$

Circuit	ABCD Parameters	
	$A = 1$ $C = 0$	$B = Z$ $D = 1$
	$A = 1$ $C = Y$	$B = 0$ $D = 1$
	$A = \cos \beta \ell$ $C = jY_0 \sin \beta \ell$	$B = jZ_0 \sin \beta \ell$ $D = \cos \beta \ell$

Linie de transmisie cu impedanta de terminatie

$$\begin{aligned}
 Z_{\text{in}} &= Z_0 \frac{(Z_L + Z_0)e^{j\beta\ell} + (Z_L - Z_0)e^{-j\beta\ell}}{(Z_L + Z_0)e^{j\beta\ell} - (Z_L - Z_0)e^{-j\beta\ell}} \\
 &= Z_0 \frac{Z_L \cos \beta\ell + jZ_0 \sin \beta\ell}{Z_0 \cos \beta\ell + jZ_L \sin \beta\ell} \\
 &= Z_0 \frac{Z_L + jZ_0 \tan \beta\ell}{Z_0 + jZ_L \tan \beta\ell} .
 \end{aligned}$$

scurtcircuit

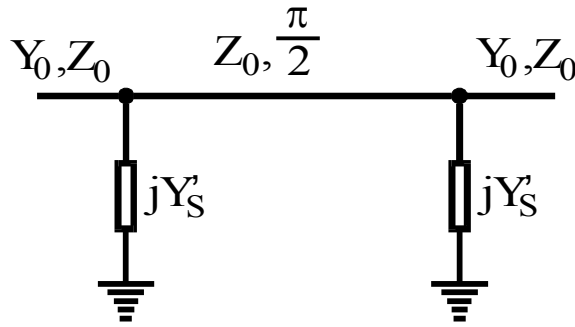
$$Z_{\text{in}} = jZ_0 \tan \beta\ell,$$

gol

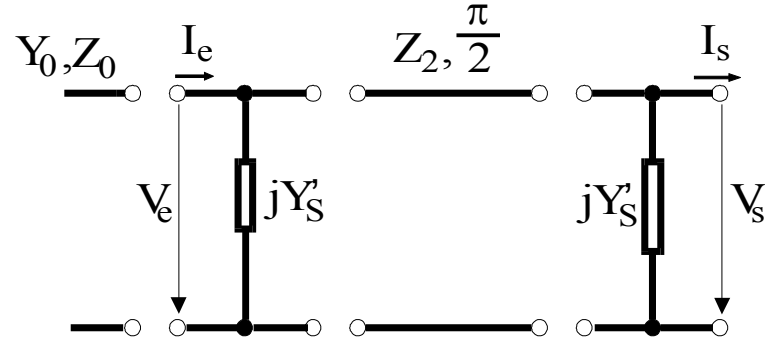
$$Z_{\text{in}} = -jZ_0 \cot \beta\ell,$$

Calculul cuploarelor cu două trepte

$$Y'_s = \begin{cases} Y_1 & \text{pentru modul par} \\ -Y_1 & \text{pentru modul impar} \end{cases}$$



a)



b)

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} 1 & 0 \\ jY'_s & 1 \end{bmatrix} \cdot \begin{bmatrix} 0 & jZ_2 \\ jY_2 & 0 \end{bmatrix} \cdot \begin{bmatrix} 1 & 0 \\ jY'_s & 1 \end{bmatrix} \cdot \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

$$\begin{bmatrix} V_e \\ I_e \end{bmatrix} = \begin{bmatrix} -Y'_s Z_2 & jZ_2 \\ -jY'^2_s Z_2 + jY_2 & -Y'_s Z_2 \end{bmatrix} \cdot \begin{bmatrix} V_s \\ I_s \end{bmatrix}$$

$$S_{11} = \frac{j \frac{Z_2}{Z_0} - Z_0 (-jY'^2_s Z_2 + jY_2)}{-2Y'_s Z_2 + j \frac{Z_2}{Z_0} + Z_0 (-jY'^2_s Z_2 + jY_2)}$$

$$S_{12} = \frac{2 \left| (-Y'_s Z_2)^2 - jZ_2 (-jY'^2_s Z_2 + jY_2) \right|}{-2Y'_s Z_2 + j \frac{Z_2}{Z_0} + Z_0 (-jY'^2_s Z_2 + jY_2)}$$

$$\Gamma = S_{11} = \frac{j(z_2 - y_2 + y'^2_s z_2)}{-2y'_s z_2 + j(z_2 + y_2 - y'^2_s z_2)} = S_{22}$$

$$S_{21} = \frac{2}{-2Y'_s Z_2 + j \frac{Z_2}{Z_0} + Z_0 (-jY'^2_s Z_2 + jY_2)} \quad S_{22} = \frac{j \frac{Z_2}{Z_0} - Z_0 (-jY'^2_s Z_2 + jY_2)}{-2Y'_s Z_2 + j \frac{Z_2}{Z_0} + Z_0 (-jY'^2_s Z_2 + jY_2)}$$

$$T = S_{21} = \frac{2}{-2y'_s z_2 + j(z_2 + y_2 - y'^2_s z_2)} = S_{12}$$

Legatura dintre parametrii S si parametrii ABCD

$$A = \sqrt{\frac{Z_{01}}{Z_{02}}} \frac{(1 + S_{11} - S_{22} - \Delta S)}{2S_{21}}$$

$$B = \sqrt{Z_{01}Z_{02}} \frac{(1 + S_{11} + S_{22} + \Delta S)}{2S_{21}}$$

$$C = \frac{1}{\sqrt{Z_{01}Z_{02}}} \frac{1 - S_{11} - S_{22} + \Delta S}{2S_{21}}$$

$$D = \sqrt{\frac{Z_{02}}{Z_{01}}} \frac{1 - S_{11} + S_{22} - \Delta S}{2S_{21}}$$

$$\Delta S = S_{11}S_{22} - S_{12}S_{21}$$

$$S_{11} = \frac{AZ_{02} + B - CZ_{01}Z_{02} - DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{12} = \frac{2(AD - BC)\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{21} = \frac{2\sqrt{Z_{01}Z_{02}}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

$$S_{22} = \frac{-AZ_{02} + B - CZ_{01}Z_{02} + DZ_{01}}{AZ_{02} + B + CZ_{01}Z_{02} + DZ_{01}}$$

Adaptarea cuplorului si coeficientul de cuplaj

$$\Gamma_e = \frac{j \cdot (z_2 - y_2 + y_1^2 z_2)}{-2y_1 z_2 + j(z_2 + y_2 - y_1^2 z_2)}$$

$$b_1 = \frac{\Gamma_e + \Gamma_o}{2} = \frac{z_2^2 - (y_2 - y_1^2 z_2)^2}{(2y_1 z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$\Gamma_o = \frac{j \cdot (z_2 - y_2 + y_1^2 z_2)}{2y_1 z_2 + j(z_2 + y_2 - y_1^2 z_2)}$$

$$b_2 = \frac{T_e + T_o}{2} = \frac{-2j(z_2 + y_2 - y_1^2 z_2)}{(2y_1 z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$T_e = \frac{2}{-2y_1 z_2 + j \cdot (z_2 + y_2 - y_1^2 z_2)}$$

$$b_3 = \frac{T_e - T_o}{2} = \frac{-4y_1 z_2}{(2y_1 z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$T_o = \frac{2}{2y_1 z_2 + j \cdot (z_2 + y_2 - y_1^2 z_2)}$$

$$b_4 = \frac{\Gamma_e - \Gamma_o}{2} = \frac{-2jy_1 z_2 (z_2 - y_2 + y_1^2 z_2)}{(2y_1 z_2)^2 + (z_2 + y_2 - y_1^2 z_2)^2}$$

$$b_1 = 0 \Rightarrow z_2 - y_2 + y_1^2 z_2 = 0 \Rightarrow z_2^2 = \frac{1}{1 + y_1^2}$$

$$y_2^2 = 1 + y_1^2$$

$$C = 10 \log \frac{P_1}{P_3} = -20 \log |b_3|, dB$$

$$b_1 = 0 \quad b_4 = 0 \quad b_3 = -y_1 z_2 \quad b_2 = -j z_2$$

$$b_3 = -\frac{\sqrt{y_2^2 - 1}}{y_2}, \quad b_2 = -\frac{j}{y_2}$$

$$C = \frac{\sqrt{y_2^2 - 1}}{y_2}$$

$$b_3 = -C$$

$$b_2 = -j\sqrt{1 - C^2}$$

$$[S] = \begin{bmatrix} 0 & -j\sqrt{1 - C^2} & -C & 0 \\ -j\sqrt{1 - C^2} & 0 & 0 & -C \\ -C & 0 & 0 & -j\sqrt{1 - C^2} \\ 0 & -C & -j\sqrt{1 - C^2} & 0 \end{bmatrix}$$

Exemplu

Proiectați un cuplor în scară pe impedanța caracteristică de $50\ \Omega$, și reprezentați mărimea parametrilor S între

$$0.5f_0 \text{ și } 1.5f_0, \text{ unde } f_0$$

este frecvența de proiectare la care liniile cuplorului sunt de lungime $\lambda/4$

Solutie

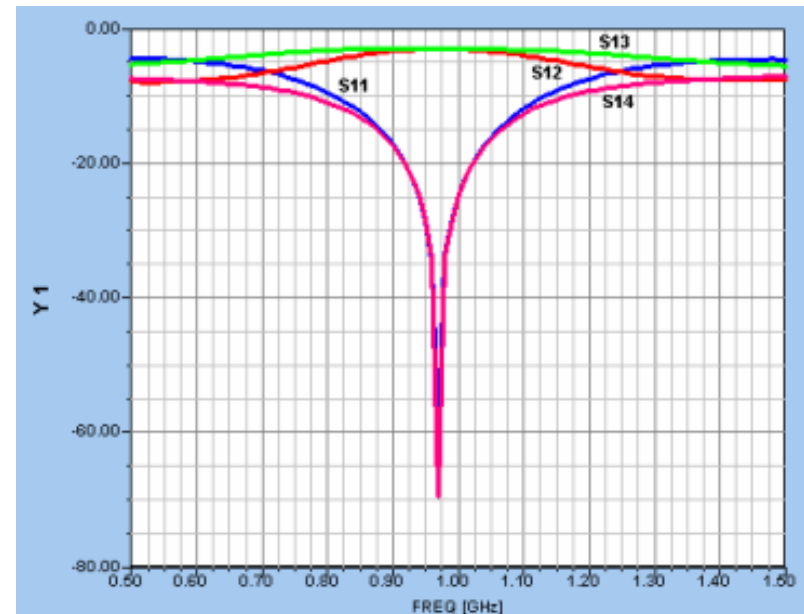
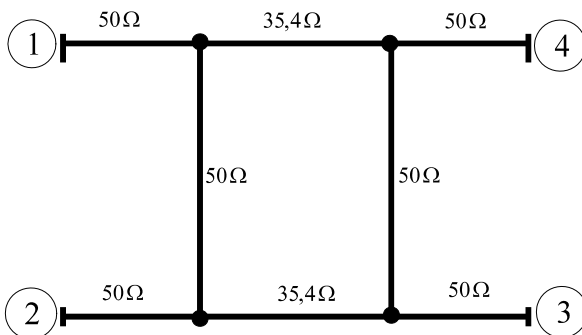
Un cuplor în scară cu $C = 3\text{dB}$, are $C = 1/\sqrt{2}$

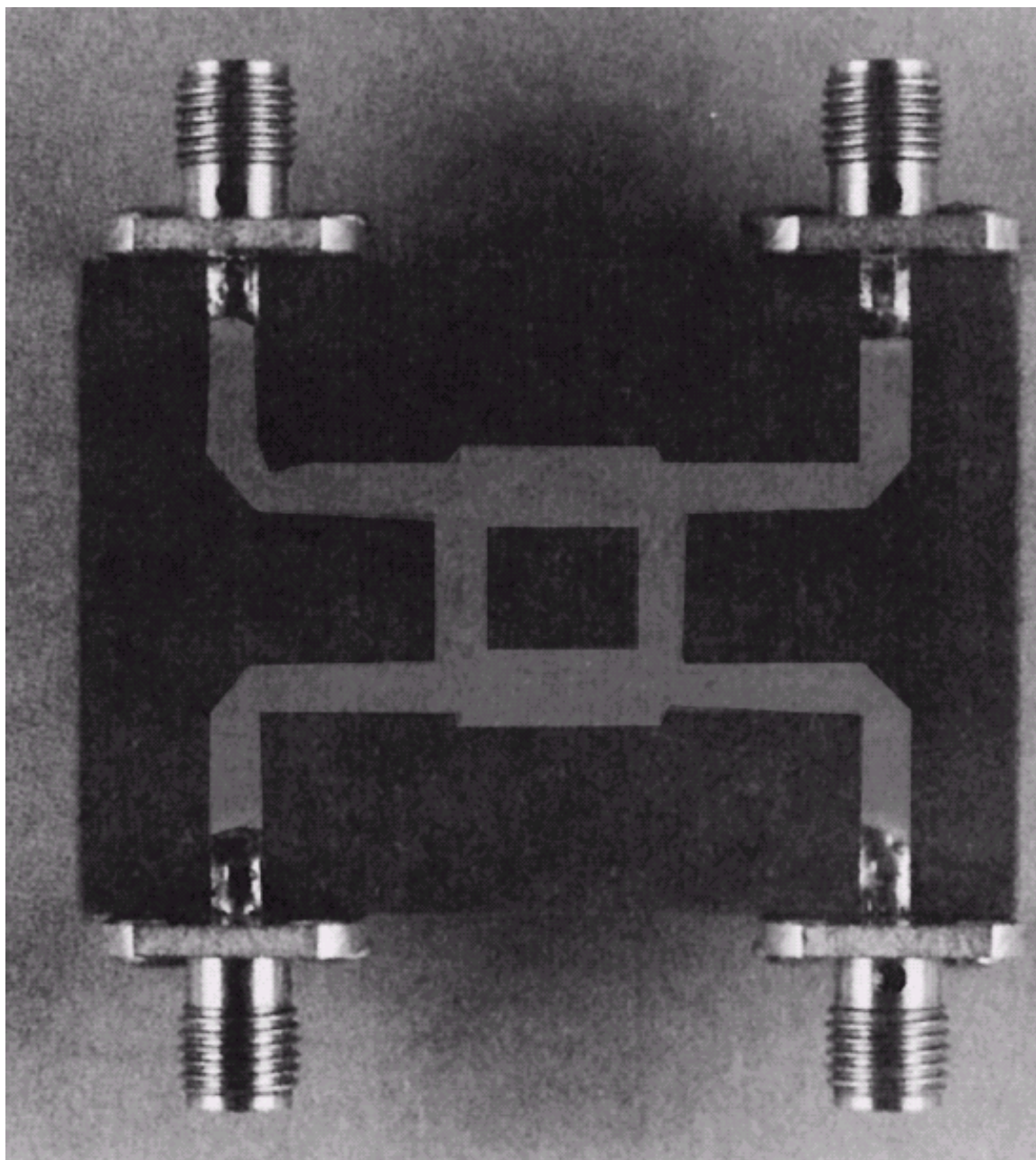
. Atunci $y_2 = \sqrt{2}$ și $y_1 = 1$

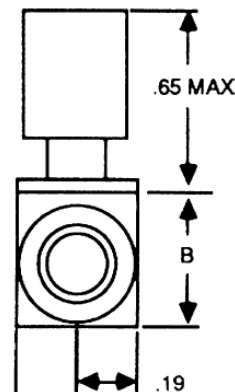
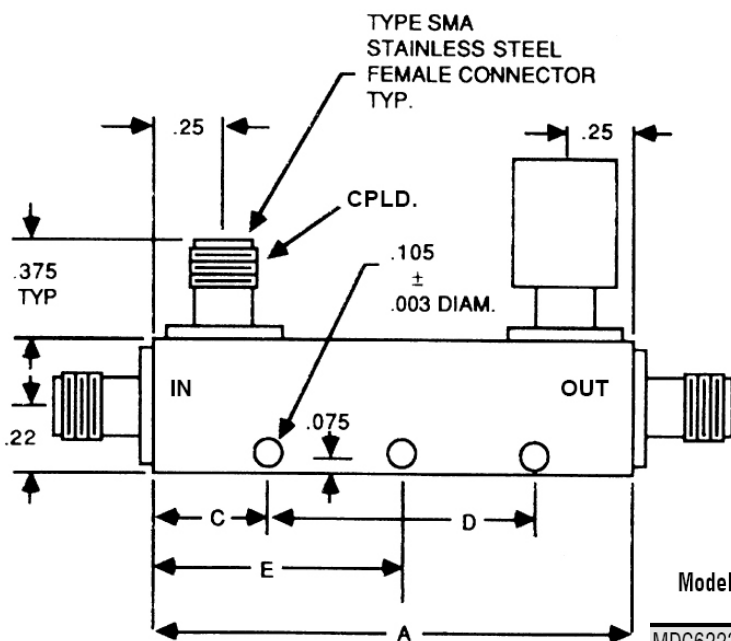
. Astfel matricea S din relația (&.47) devine cea din relația (&.38). În plus, pentru $Z_0 = 50\Omega$

, impedanțele caracteristice ale liniilor cuplorului vor fi:

$$Z_1 = Z_0 = 50\Omega \quad Z_2 = \frac{Z_0}{\sqrt{2}} = 35.4\Omega$$

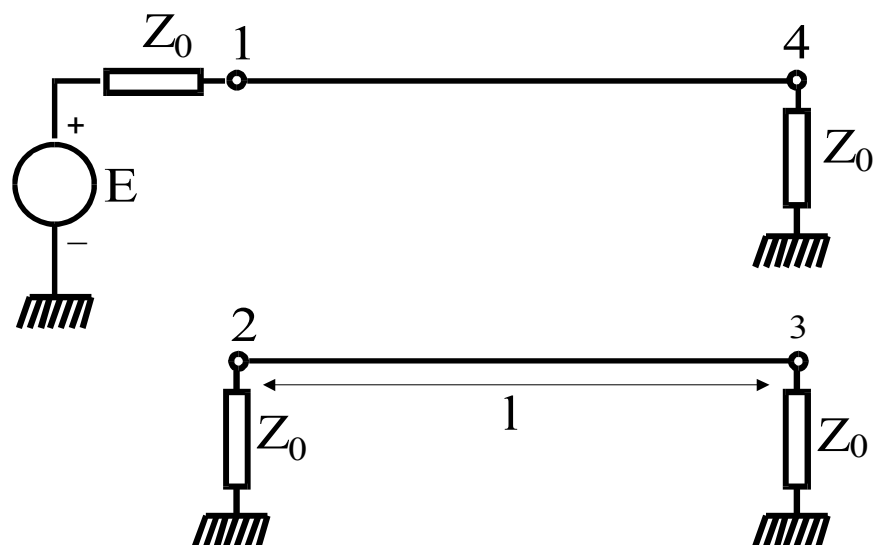
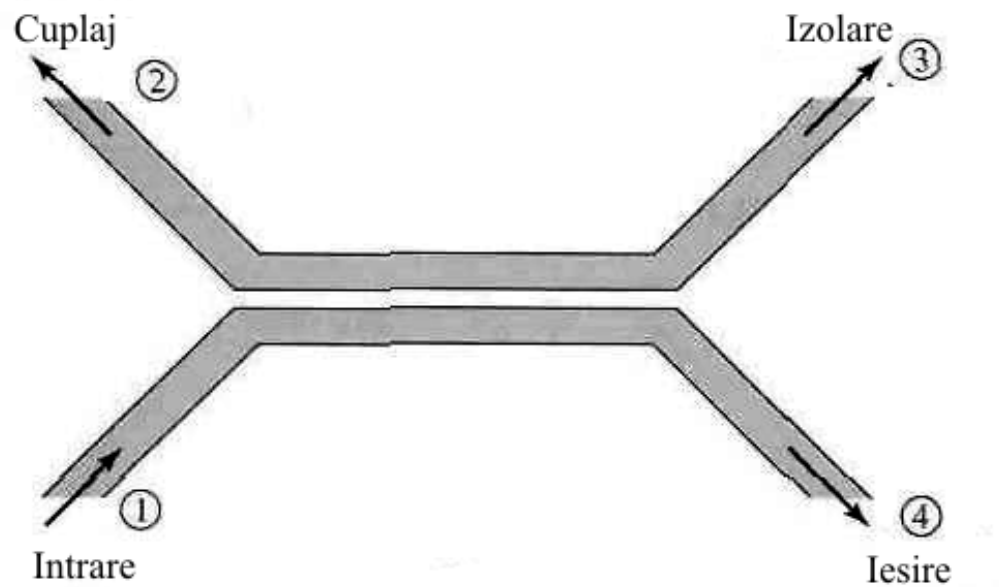




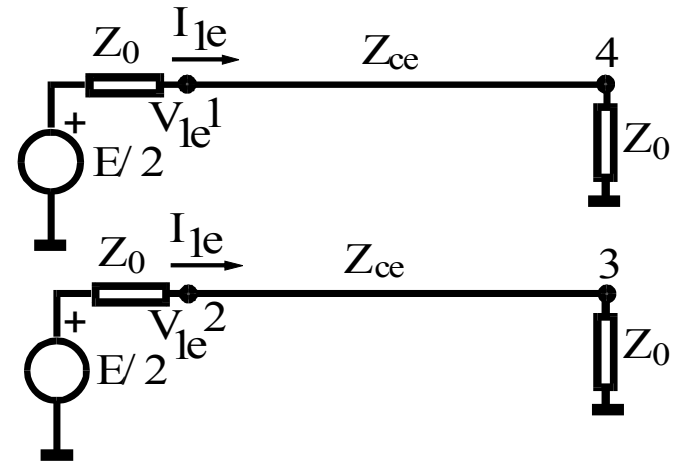
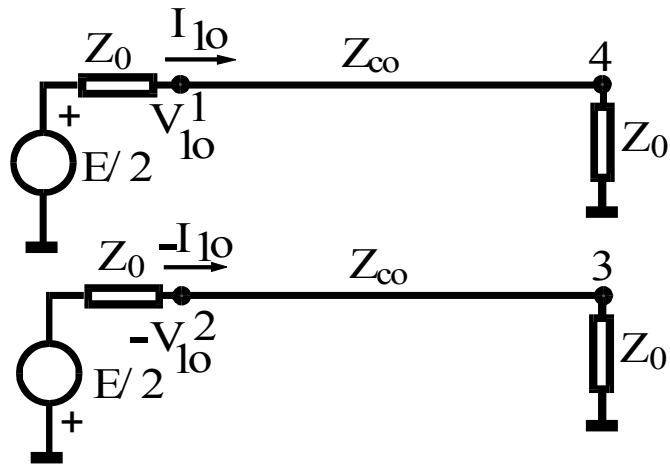


Model No.	Frequency Range (Ghz)	Coupling † (dB)	Freq. Sens. (dB)	Insertion Loss (dB)		Directivity (dB min.)	VSWR max.	
				Excl. Cpld Pwr	True		Primary Line	Secondary Line
MDC6223-6	0.5-1.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6223-10	0.5-1.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6223-20	0.5-1.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6223-30	0.5-1.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-6	1.0-2.0	6 ±1.00	±0.60	0.20	1.80	25	1.15	1.15
MDC6224-10	1.0-2.0	10 ±1.25	±0.75	0.20	0.80	25	1.10	1.10
MDC6224-20	1.0-2.0	20 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6224-30	1.0-2.0	30 ±1.25	±0.75	0.15	0.20	25	1.10	1.10
MDC6225-6	2.0-4.0	6 ±1.00	±0.60	0.20	1.80	22	1.15	1.15
MDC6225-10	2.0-4.0	10 ±1.25	±0.75	0.20	0.80	22	1.15	1.15
MDC6225-20	2.0-4.0	20 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6225-30	2.0-4.0	30 ±1.25	±0.75	0.15	0.20	22	1.15	1.15
MDC6266-6	2.6-5.2	6 ±1.00	±0.60	0.20	1.80	20	1.25	1.25
MDC6266-10	2.6-5.2	10 ±1.25	±0.75	0.20	0.80	20	1.25	1.25
MDC6266-20	2.6-5.2	20 ±1.25	±0.75	0.20	0.25	20	1.25	1.25
MDC6266-30	2.6-5.2	30 ±1.25	±0.75	0.20	0.20	20	1.25	1.25
MDC6226-6	4.0-8.0	6 ±1.00	±0.60	0.25	1.90	20	1.25	1.25
MDC6226-10	4.0-8.0	10 ±1.25	±0.75	0.25	0.90	20	1.25	1.25
MDC6226-20	4.0-8.0	20 ±1.25	±0.75	0.25	0.30	20	1.25	1.25
MDC6226-30	4.0-8.0	30 ±1.25	±0.75	0.25	0.25	20	1.25	1.25
MDC6227-6	7.0-12.4	6 ±1.00	±0.50	0.30	2.00	17	1.30	1.30
MDC6227-10	7.0-12.4	10 ±1.00	±0.50	0.30	1.00	17	1.30	1.30
MDC6227-20	7.0-12.4	20 ±1.00	±0.50	0.30	0.35	17	1.30	1.30

Cuplorul prin proximitate

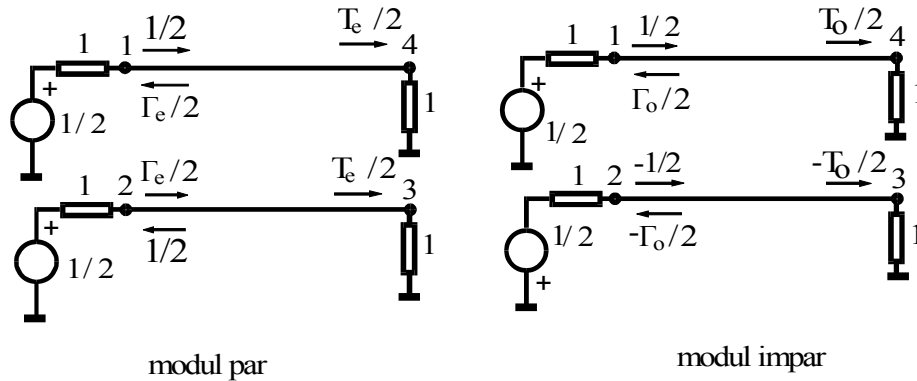


Adaptarea cuplului prin proximitate



$$\begin{cases} Z_{ce}Z_{co} = Z_0^2 \\ \theta_e = \theta_o \end{cases}$$

Directivitatea și coeficientul de cuplaj ale cuplorului prin proximitate



$$a_1 = a_{1e} + a_{1o} = 1, a_2 = a_3 = a_4 = 0$$

$$b_1 = \frac{1}{2}(\Gamma_e + \Gamma_o) = 0 \Leftrightarrow$$

$$b_2 = \frac{1}{2}(\Gamma_e - \Gamma_o) = \frac{jC \sin(\theta)}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$b_3 = \frac{1}{2}(T_e - T_o) = 0$$

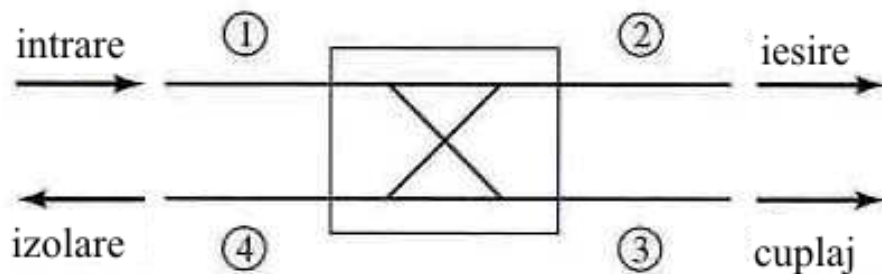
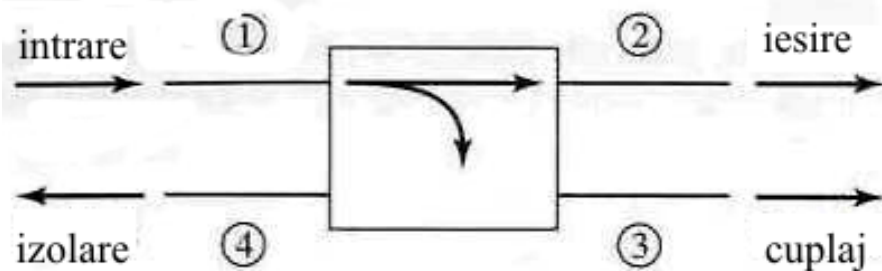
$$b_4 = \frac{1}{2}(T_e + T_o) = \frac{\sqrt{1-C^2}}{\cos(\theta)\sqrt{1-C^2} + j\sin(\theta)}$$

$$C = \frac{Z_{ce} - Z_{co}}{Z_{ce} + Z_{co}}$$

$$\theta = \pi/2$$

$$[S] = \begin{bmatrix} 0 & C & 0 & -j\sqrt{1-C^2} \\ C & 0 & -j\sqrt{1-C^2} & 0 \\ 0 & -j\sqrt{1-C^2} & 0 & C \\ -j\sqrt{1-C^2} & 0 & C & 0 \end{bmatrix}$$

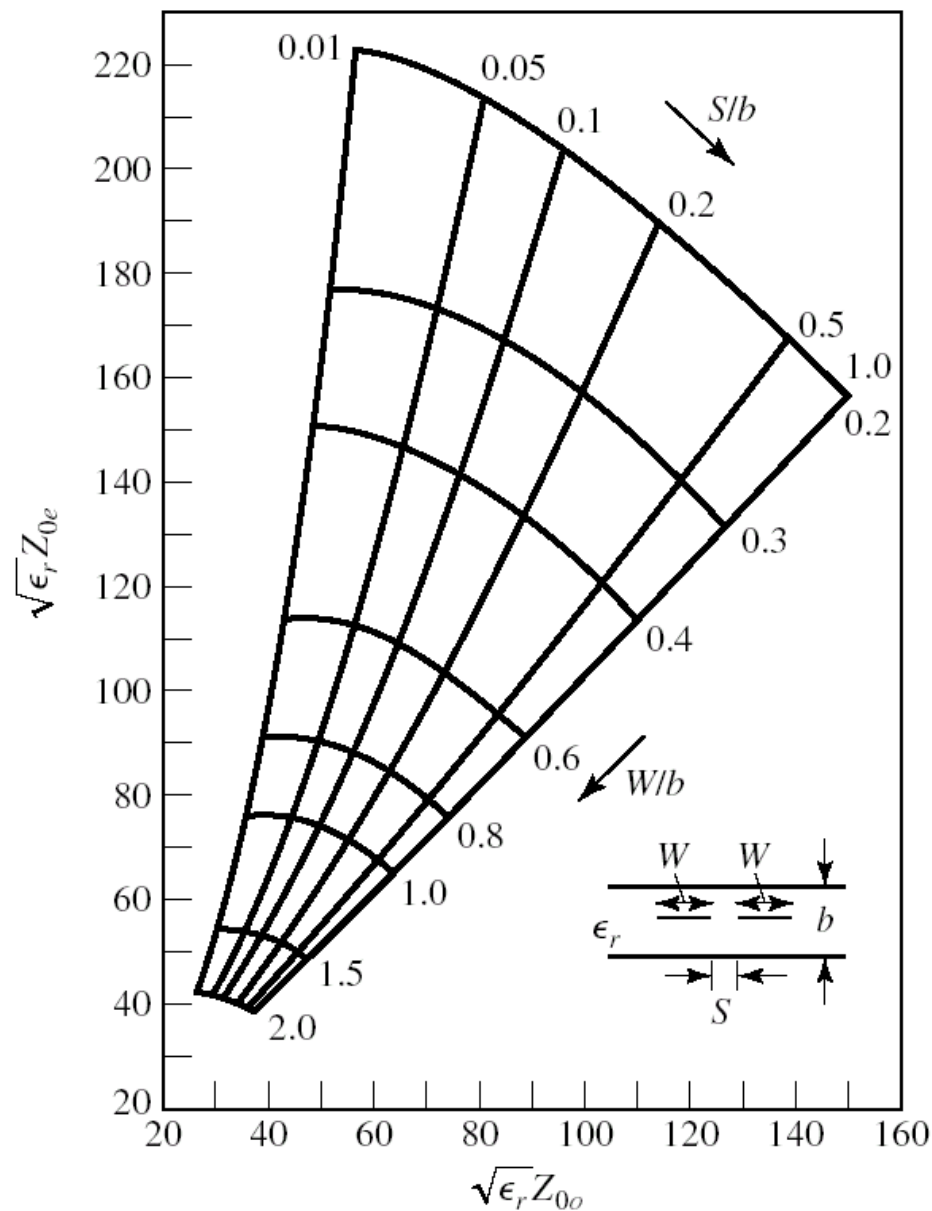
Cuplor prin proximitate



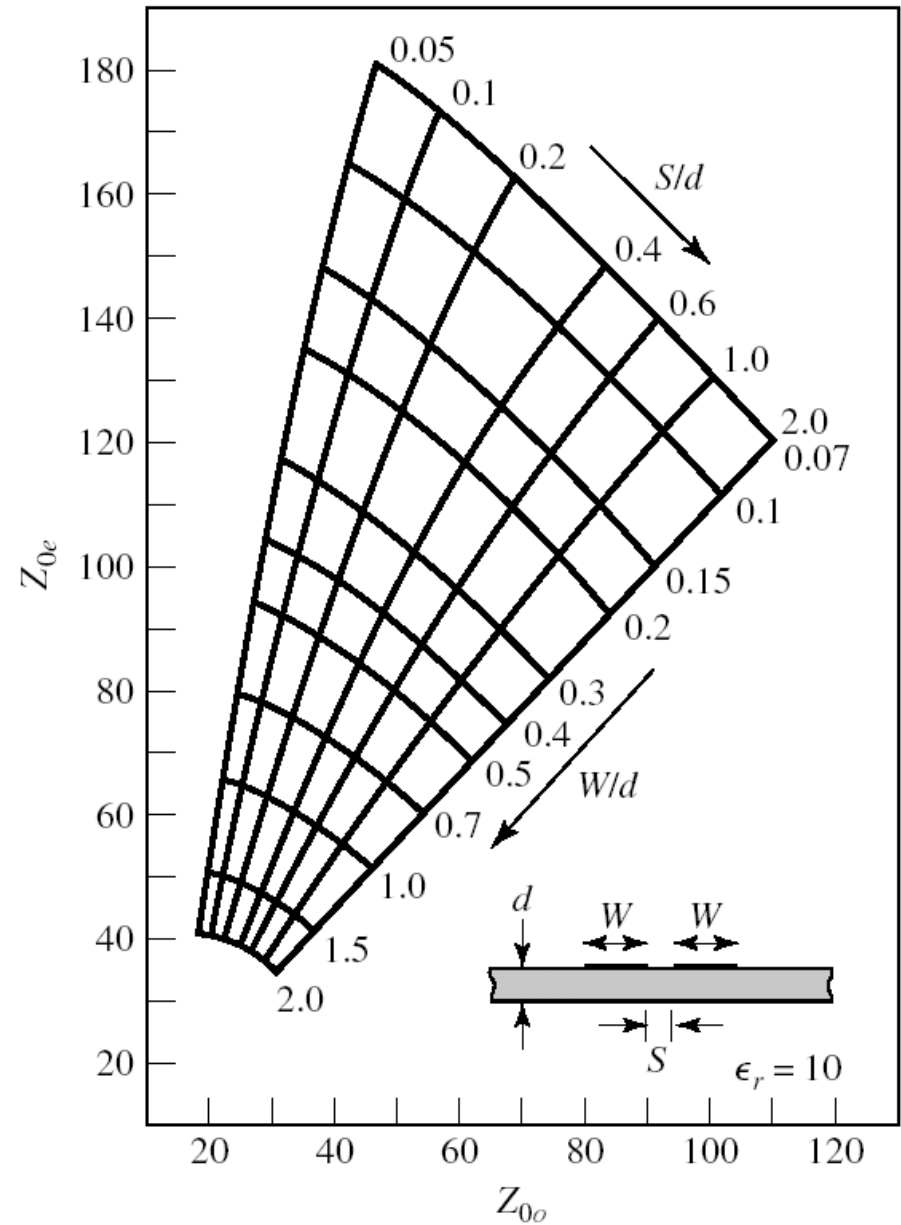
$$[S] = -j \cdot \begin{bmatrix} 0 & \sqrt{1-C^2} & jC & 0 \\ \sqrt{1-C^2} & 0 & 0 & jC \\ jC & 0 & 0 & \sqrt{1-C^2} \\ 0 & jC & \sqrt{1-C^2} & 0 \end{bmatrix}$$

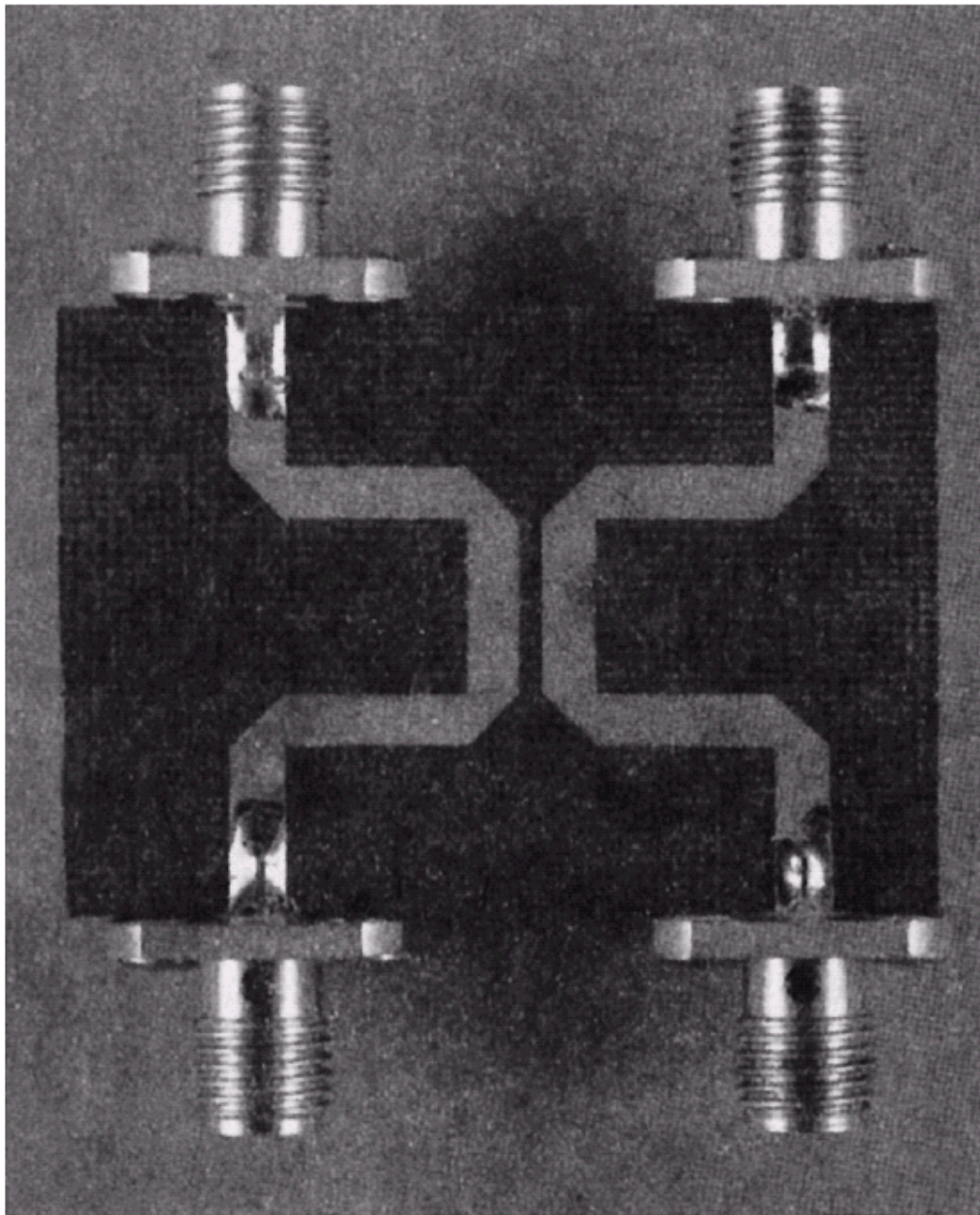
$$[S] = \frac{1}{\sqrt{2}} \begin{bmatrix} 0 & 1 & j & 0 \\ 1 & 0 & 0 & j \\ j & 0 & 0 & 1 \\ 0 & j & 1 & 0 \end{bmatrix}$$

Normalized even- and odd-mode characteristic impedance design data for edge-coupled striplines.



Even- and odd-mode characteristic impedance design data for coupled microstrip lines on a substrate with $\epsilon_r = 10$.





Exemplu

Proiectați un cuplor prin proximitate de 20 dB, în tehnologie stripline, folosind o distanță între planele de masă de 0.158 cm și cu o permitivitate electrică relativă de 2.56, pe o impedanță de $50\ \Omega$, la frecvența de 3 GHz. Reprezentați cuplajul și directivitatea între 1 și 5 GHz.

Soluție

$$C = 10^{-20/20} = 0.1$$

$$Z_{co} = 50 \sqrt{\frac{0.9}{1.1}} = 45.23 \Omega$$

$$Z_{ce} = 50 \sqrt{\frac{1.1}{0.9}} = 55.28 \Omega$$

TRL - Edge-coupled Symmetric Stripline (CPL)1

File Edit View Structure Window Help

Edge-coupled Symmetric Stripline (CPL)1

Dimensions

W 1.14072

S 0.51747

P 15.6142

Electrical

Z0 50

K 20

E 90

Zo 45.2267

Ze 55.2771

Units

Dimension mm

Frequency GHz

Impedance Ohm

Electrical Length Deg

Resistivity uOhm*cm

Frequency 3 Analysis Auto Calculate Off ! Reset All ! Synthesis 3

Substrate

Required

B 1.58 ER 2.56

Optional

TAND 0

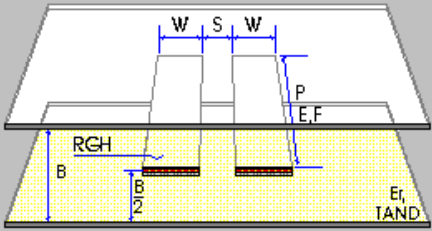
Metallization

Layers	Metal Name	Code	Resistivity	Thickness	
Bottom	*None*				Reset
Middle	*None*				Reset
Top	*None*				Reset

RGH 0 Add new metal

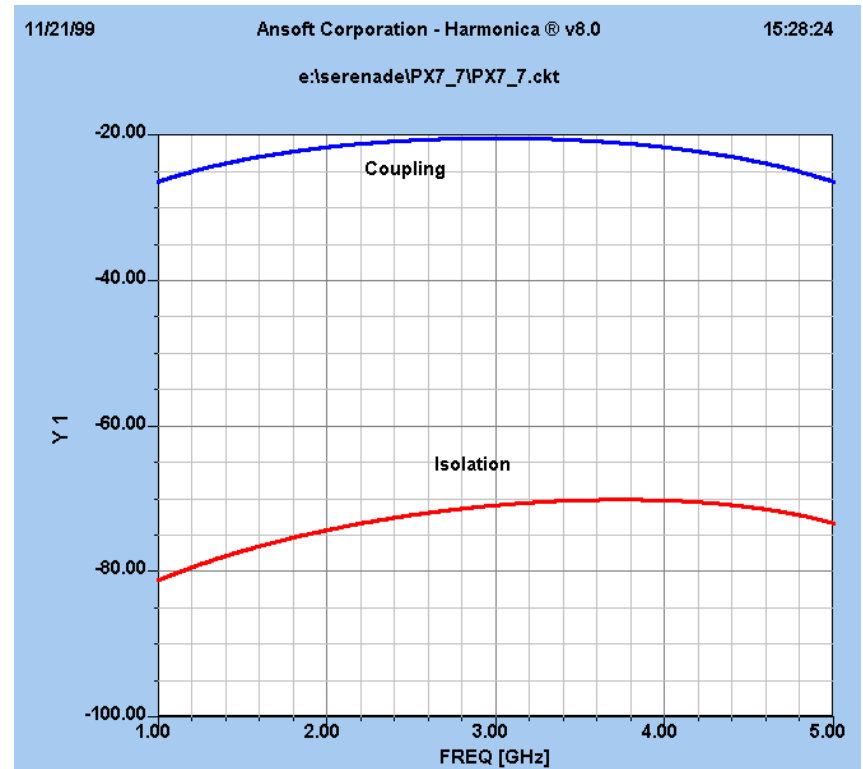
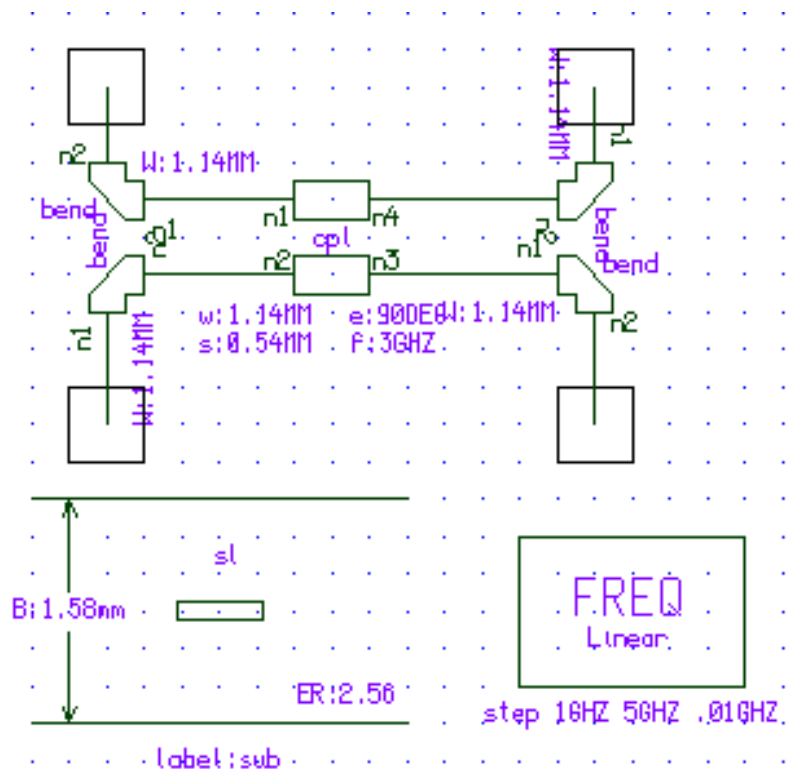
For Help, press F1

NUM

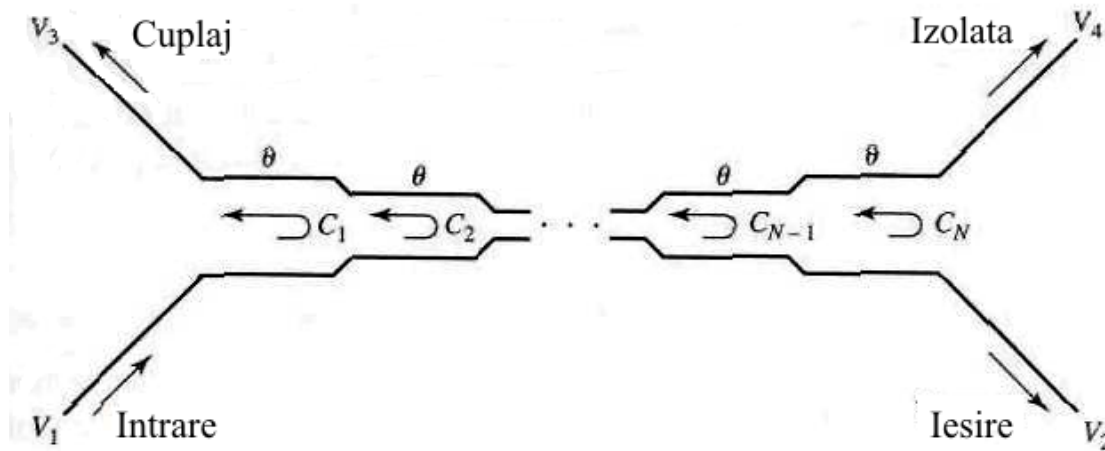


$$Z_{ce} = Z_0 \sqrt{\frac{1+C}{1-C}}, \quad Z_{co} = Z_0 \sqrt{\frac{1-C}{1+C}}$$

Simulare



Cuplor prin proximitate cu mai multe secțiuni



$$C \ll 1$$

$$\frac{V_3}{V_1} = b_3 = \frac{jC \sin \theta}{\cos \theta \sqrt{1-C^2} + j \sin \theta} = \frac{jC \operatorname{tg} \theta}{\sqrt{1-C^2} + j \operatorname{tg} \theta} \approx \frac{jC \operatorname{tg} \theta}{1 + j \operatorname{tg} \theta} = jC \sin \theta e^{-j\theta}$$

$$\frac{V_2}{V_1} = b_2 = \frac{\sqrt{1-C^2}}{\cos \theta \sqrt{1-C^2} + j \sin \theta} \approx \frac{1}{\cos \theta + j \sin \theta} = e^{-j\theta}$$

$$C = \frac{V_3}{V_1} = 2j \sin \theta e^{-j\theta} e^{-j(N-1)\theta} \left[C_1 \cos(N-1)\theta + C_2 \cos(N-3)\theta + \dots + \frac{1}{2} C_{\frac{N+1}{2}} \right]$$

Exemplu

Să se proiecteze un cuplor cu trei secțiuni, avînd un cuplaj de 20 dB, cu caracteristică binomială (maxim plat), pe o impedanță de $50\ \Omega$, la frecvența centrală de 3 GHz. Să se reprezinte grafic cuplajul și directivitatea între 1 și 5 GHz.

Solutie

$$(N = 3) \quad C_0 = 20 \text{ dB} \quad \theta = \pi/2$$

$$\left. \frac{d^n}{d\theta^n} C(\theta) \right|_{\theta=\pi/2} = 0, n = 1, 2$$

$$C = \left| \frac{V_3}{V_1} \right| = 2 \sin \theta \left[C_1 \cos 2\theta + \frac{1}{2} C_2 \right] = C_1 (\sin 3\theta - \sin \theta) + C_2 \sin \theta$$

$$\frac{dC}{d\theta} = [3C_1 \cos 3\theta + (C_2 - C_1) \cos \theta] \big|_{\theta=\pi/2} = 0$$

$$\frac{d^2 C}{d\theta^2} = [-9C_1 \sin 3\theta - (C_2 - C_1) \sin \theta] \big|_{\theta=\pi/2} = 10C_1 - C_2 = 0$$

$$\begin{cases} C_2 - 2C_1 = 0.1 \\ 10C_1 - C_2 = 0 \end{cases}$$

$$\begin{cases} C_1 = C_3 = 0.0125 \\ C_2 = 0.125 \end{cases}$$

$$Z_{0e}^1 = Z_{0e}^3 = 50 \sqrt{\frac{1.0125}{0.9875}} = 50.63 \Omega$$

$$Z_{0o}^1 = Z_{0o}^3 = 50 \sqrt{\frac{0.9875}{1.0125}} = 49.38 \Omega$$

$$Z_{0e}^2 = 50 \sqrt{\frac{1.125}{0.875}} = 56.69 \Omega$$

$$Z_{0o}^2 = 50 \sqrt{\frac{0.875}{1.125}} = 44.10 \Omega$$

Simulare

