

Optoelectronică, structuri și tehnologii

Curs 2
2012/2013

Lumina ca undă electromagnetică

Capitolul 2

Ecuatiile lui Maxwell

$$\nabla \times E = -\frac{\partial B}{\partial t}$$

$$\nabla \times H = \frac{\partial D}{\partial t} + J$$

$$\nabla \cdot D = \rho$$

$$\nabla \cdot B = 0$$

$$\nabla \cdot J = -\frac{\partial \rho}{\partial t}$$

▶ Ecuatii constitutive

$$D = \varepsilon \cdot E$$

$$B = \mu \cdot H$$

$$J = \sigma \cdot E$$

▶ In vid

$$\mu_0 = 4\pi \times 10^{-7} \text{ H/m}$$

$$\varepsilon_0 = 8,854 \times 10^{-12} \text{ F/m}$$

$$c_0 = \frac{1}{\sqrt{\varepsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ m/s}$$

Câmpuri electromagnetice cu variație armonică în timp

$$X = X_0 e^{j \cdot \omega \cdot t} \quad \frac{\partial X}{\partial t} = j \cdot \omega \cdot X \quad g(\omega) = \int_{-\infty}^{\infty} f(t) \cdot e^{-j\omega t} dt \quad f(t) = \int_{-\infty}^{\infty} g(\omega) \cdot e^{j\omega t} d\omega$$

► Simplificarea ecuațiilor lui Maxwell

$$\nabla^2 E + \omega^2 \epsilon \mu E = j\omega \mu J + \frac{1}{\epsilon} \nabla \rho$$

$$\nabla^2 H + \omega^2 \epsilon \mu H = -\nabla \times J$$

$$\nabla \cdot E = \frac{\rho}{\epsilon}$$

$$\nabla \cdot H = 0$$

► Ecuațiile Helmholtz sau ecuațiile de propagare

Mediu lipsit de sarcini electrice

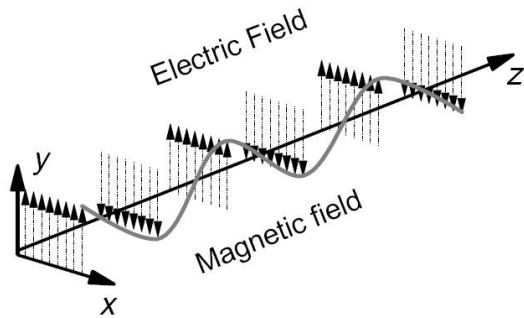
$$\nabla^2 E - \gamma^2 E = 0$$

$$\nabla^2 H - \gamma^2 H = 0$$

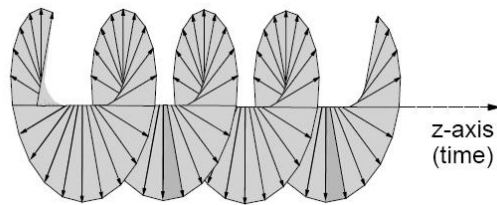
$$\gamma^2 = -\omega^2 \epsilon \mu + j\omega \mu \sigma$$

γ – Constanta de propagare

Solutia ecuatiilor de propagare



Propagare



Polarizare circulara

Camp electric dupa directia Oy,
propagare dupa directia Oz

$$E_y = E_+ e^{-\gamma \cdot z} + E_- e^{\gamma \cdot z}$$

$$\gamma = \sqrt{-\omega^2 \epsilon \mu + j \omega \mu \sigma} = \alpha + j \cdot \beta$$

Exista numai unda progresiva $E_+ \Rightarrow A$

$$E_y = A e^{-(\alpha + j \cdot \beta) \cdot z}$$

Camp armonic

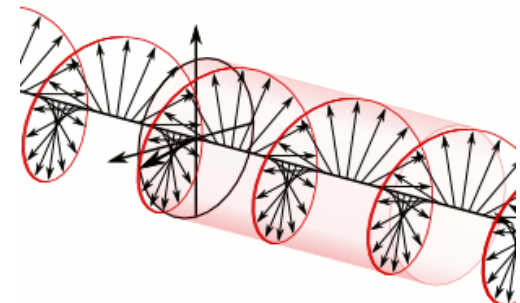
$$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega t - \beta \cdot z)}$$

Amplitudine

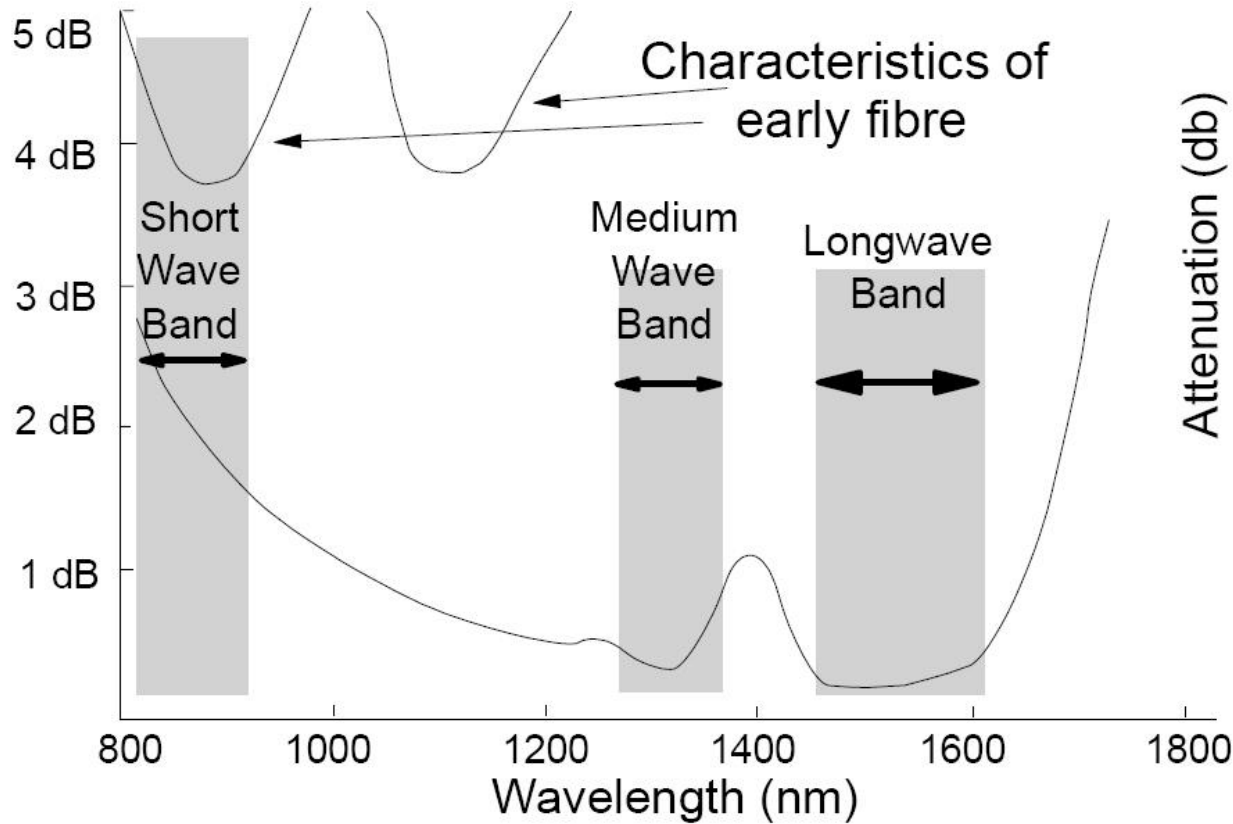
Atenuare

Propagare

(variatie in timp si spatiu)



Atenuarea pe 1 km in SiO₂



Atenuare

$$W, P \sim \int E^2$$

$$E_y(z_1) = Ct \cdot e^{-\alpha \cdot z_1} \cdot e^{j(\omega t - \beta \cdot z_1)}$$

$$E_y(z_2) = Ct \cdot e^{-\alpha \cdot z_2} \cdot e^{j(\omega t - \beta \cdot z_2)}$$

$$A = \frac{P_2}{P_1} = \frac{Ct^2 \cdot e^{-2\alpha \cdot z_2}}{Ct^2 \cdot e^{-2\alpha \cdot z_1}} = e^{-2\alpha \cdot (z_2 - z_1)}$$

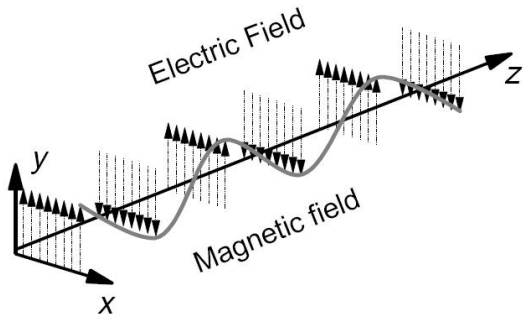
$$A[dB] = 10 \log_{10} \frac{P_2}{P_1} = 10 \log_{10} \left[e^{-2\alpha \cdot (z_2 - z_1)} \right]$$

$$A[dB] = -20 \cdot \alpha \cdot (z_2 - z_1) \log_{10} e = -8.686 \cdot \alpha \cdot (z_2 - z_1)$$

$$A / L [dB / km] = -8.686 \cdot \alpha < 0$$

- ▶ Atenuarea se exprima de obicei in **dB/km**
 - ▶ de obicei valori pozitive
 - ▶ semnul = **implicit**

Parametri de propagare



$$\nabla \times E = -j\omega\mu \cdot H$$

$$H_x = \frac{j\gamma \cdot E_y}{\omega\mu}$$

Mediu fara pierderi, $\sigma = 0$ $\gamma = j\omega \cdot \sqrt{\epsilon\mu}$

$$\eta = \frac{E_y}{H_x} = \sqrt{\frac{\mu}{\epsilon}} \quad \text{Impedanta intrinseca a mediului}$$

$E_y = A \cdot e^{-\alpha \cdot z} \cdot e^{j(\omega \cdot t - \beta \cdot z)}$ punctele de faza constanta: $(\omega \cdot t - \beta \cdot z) = \text{const}$

Viteza de faza $v = \frac{dz}{dt} = \frac{\omega}{\beta} = \frac{1}{\sqrt{\epsilon\mu}}$

Viteza de grup $v_g = \frac{dz}{dt} = \frac{d\omega}{d\beta}$ in medii dispersive unde $\beta = \beta(\omega)$

Parametri de propagare

▶ In vid

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega \quad v = v_g = c_0 \quad c_0 = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ m/s}$$

$$\lambda_0 = \frac{2\pi}{\beta} = \frac{c_0}{f} \quad T = \frac{2\pi}{\omega} = \frac{1}{f}$$

Periodicitate in spatiu

Periodicitate in timp

▶ In mediu nedispersiv ϵ_r

$$c = \frac{1}{\sqrt{\epsilon \cdot \mu_0}} = \frac{1}{\sqrt{\epsilon_0 \epsilon_r \cdot \mu_0}} = \frac{c_0}{\sqrt{\epsilon_r}}$$

$$n = \sqrt{\epsilon_r} \quad \text{Indice de refractie al mediului} \quad c = \frac{c_0}{n}$$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\lambda = \frac{2\pi}{\beta} = \frac{c}{f}$$

$$\lambda = \frac{c_0}{n \cdot f} = \frac{\lambda_0}{n}$$

Parametri, dependenta de mediu

$$\eta_0 = \sqrt{\frac{\mu_0}{\epsilon_0}} = 377\Omega$$

$$c_0 = \frac{1}{\sqrt{\epsilon_0 \cdot \mu_0}} = 2,99790 \cdot 10^8 \text{ m/s}$$

$n = 1$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\lambda_0 = \frac{2\pi}{\beta} = \frac{c_0}{f}$$

$$\eta = \frac{\eta_0}{n}$$

$$c = \frac{c_0}{n}$$

$n = \sqrt{\epsilon_r}$

$$T = \frac{2\pi}{\omega} = \frac{1}{f}$$

$$\lambda = \frac{c_0}{n \cdot f} = \frac{\lambda_0}{n}$$



Dispersia

- ▶ In medii dispersive $\beta = \beta(\omega)$, $n = n(\omega)$

$$\frac{d\beta}{d\omega} = \frac{d}{d\omega} \left(\frac{\omega \cdot n}{c} \right) = \frac{1}{c} \left(n + \omega \frac{dn}{d\omega} \right)$$

$$\frac{d\beta}{d\omega} = -\frac{\lambda}{\omega} \cdot \frac{d\beta}{d\lambda} = \frac{1}{c} \left(n - \lambda \frac{dn}{d\lambda} \right) = \tau \quad (s/m)$$

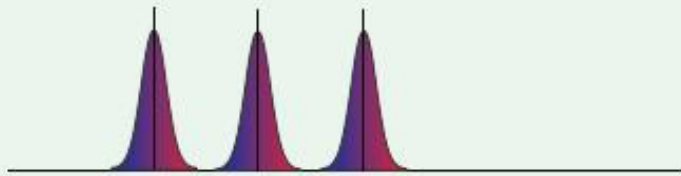
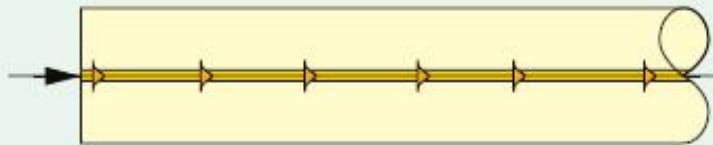
$$D = \frac{d\tau}{d\lambda} = \frac{1}{c} \left(\frac{dn}{d\lambda} - \lambda \frac{d^2n}{d\lambda^2} - \frac{dn}{d\lambda} \right) = -\frac{\lambda}{c} \frac{d^2n}{d\lambda^2} \quad (s/m^2)$$

- ▶ Dispersia se exprima de obicei in **ps/nm/km** si permite aflarea intarzierilor aparute intre moduri (latirea impulsurilor) pentru o anumita latime spectrala si o anumita distanta parcursa

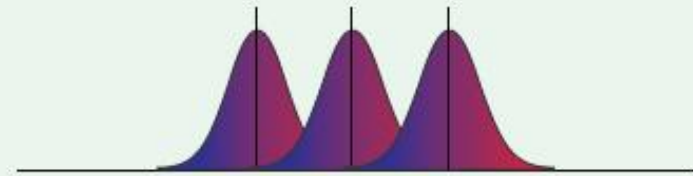
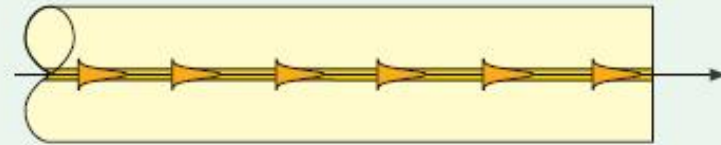
$$\Delta\tau = D \cdot \Delta\lambda \cdot L$$

Dispersie

> 50 km Single-mode step index
< 10 km Multimode graded index
< 1 km Multimode step index

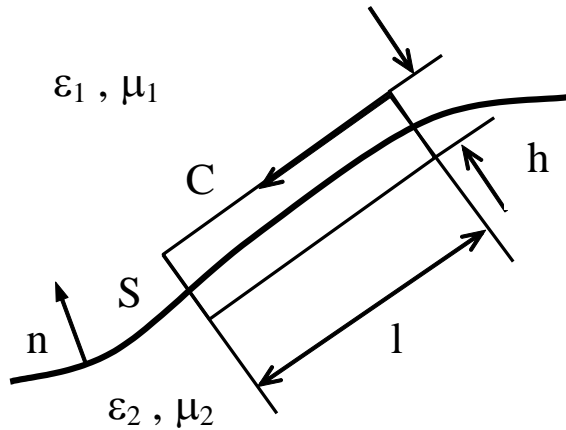


Transmission:
Well-defined pulses but not absolutely monochromatic.
Typical spectral width < 0.8 nm



Reception:
Pulse broadening caused by the laser's spectral width and the difference between the refractive indices of the red and blue ends of the light pulse.

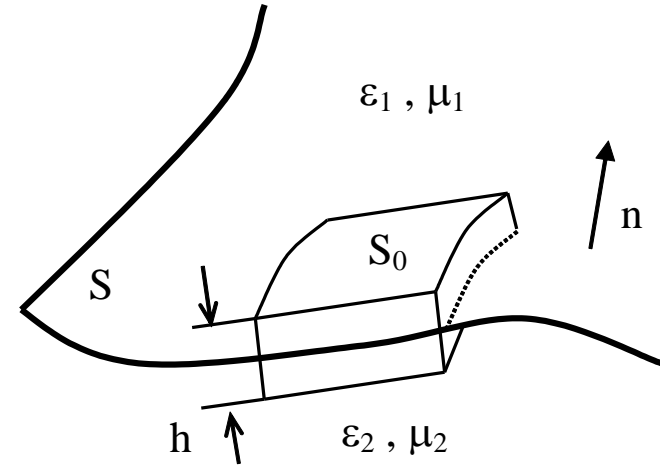
Condiții la limita de separație între două medii



a)

$$n \times (E_1 - E_2) = 0$$

$$n \times (H_1 - H_2) = J_S$$



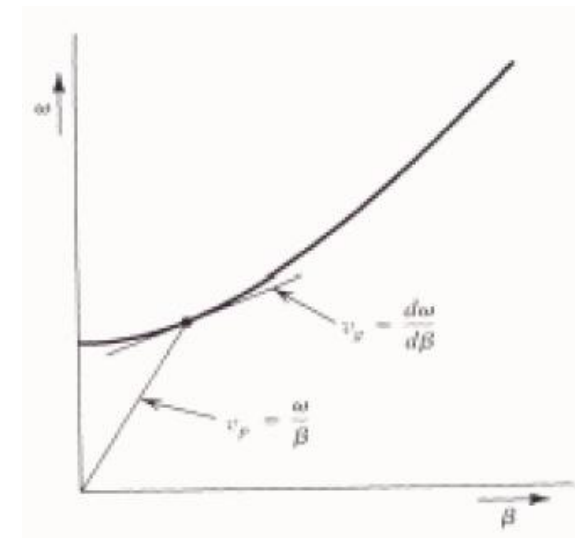
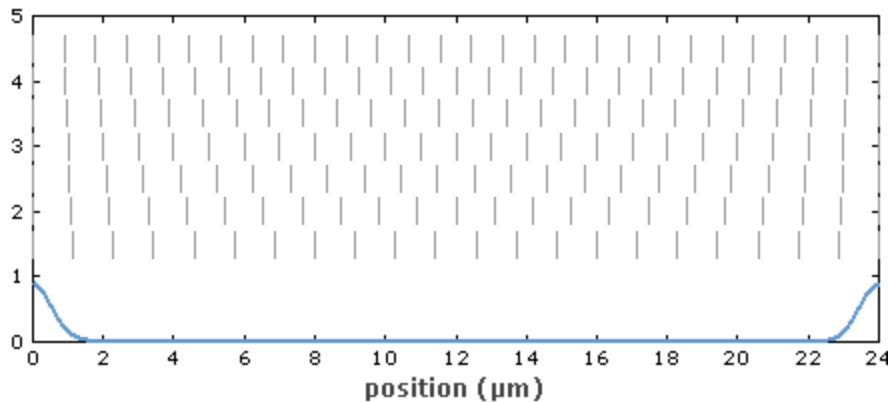
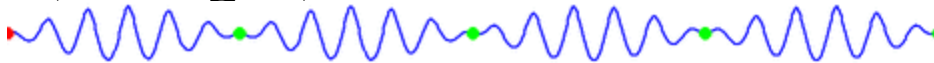
b)

$$n \cdot (D_1 - D_2) = \rho_S$$

$$n \cdot (B_1 - B_2) = 0$$

Viteze de grup si faza

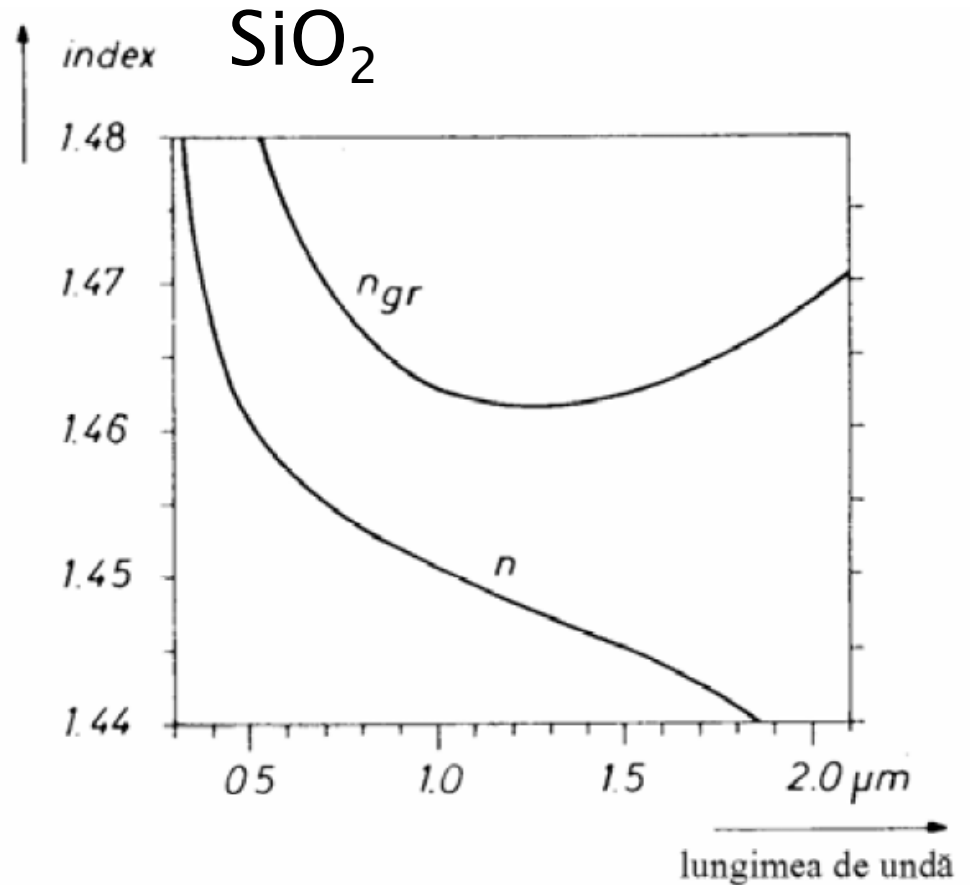
- ▶ Viteza de faza – viteza virtuala cu care circula punctul cu o anumita faza
- ▶ Viteza de grup – viteza cu care circula informatia (energia)



Dispersie normala

$$n_{gr} = n - \lambda \frac{dn}{d\lambda}$$

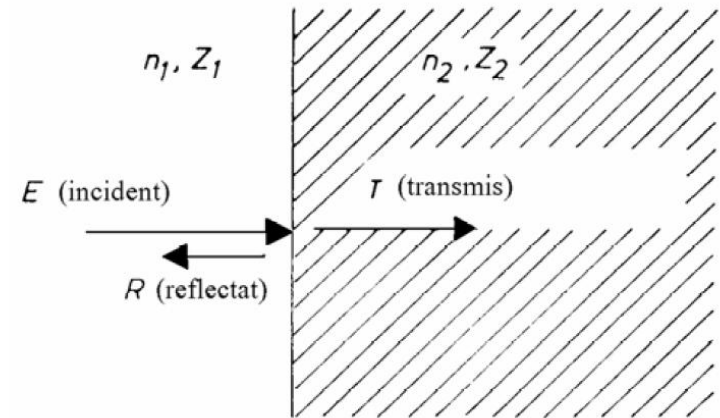
$$D = \frac{d\tau}{d\lambda} = \frac{1}{c} \cdot \frac{dn_{gr}}{d\lambda}$$



Transmisia puterii între medii

- ▶ incidenta normala
- ▶ reflexia in amplitudine

$$Z = \frac{Z_0}{n} \quad \Gamma = \frac{Z_2 - Z_1}{Z_2 + Z_1} = \frac{n_1 - n_2}{n_1 + n_2}$$



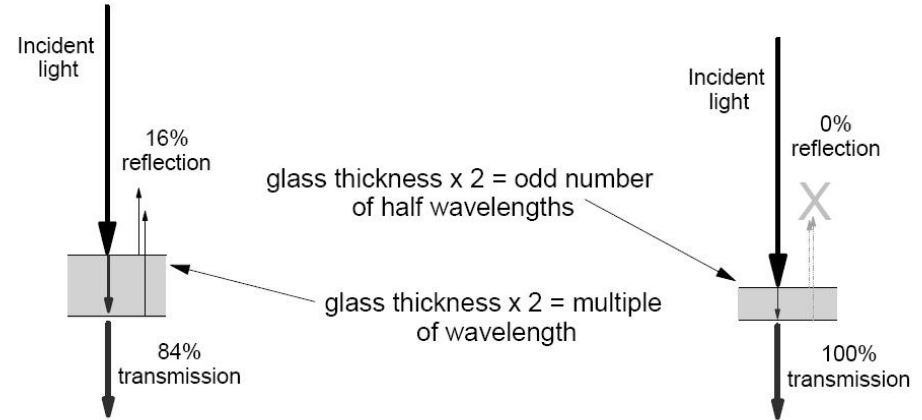
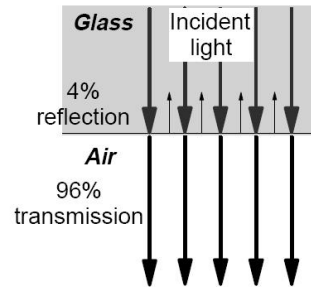
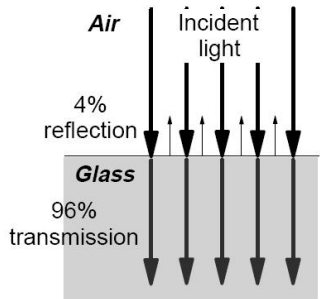
- ▶ densitatea de putere proportionala cu patratul amplitudinii câmpului

$$r = \left(\frac{n_1 - n_2}{n_1 + n_2} \right)^2 \quad t = \left(\frac{2n_1}{n_1 + n_2} \right)^2$$

- ▶ interfata aer–sticla ($n_1 = 1$, $n_2 = 1.5$)

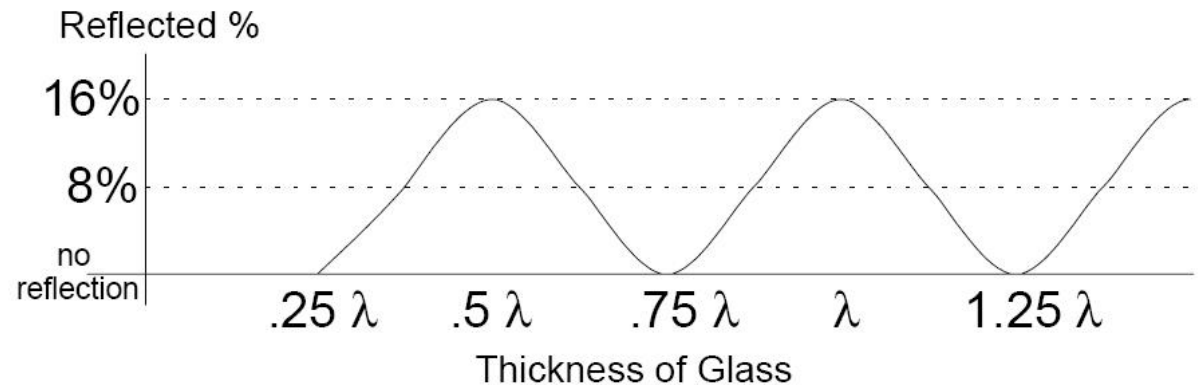
$$r = 0.04 = 4\%$$

Transmisia printr-o lamela



$$\Gamma = \frac{1.5 - 1}{1.5 + 1} = 0.2; \quad r = \Gamma^2 = 0.04 = 4% \quad \Gamma_{\max} = 0.2 + 0.2; \quad r_{\max} = \Gamma_{\max}^2 = 0.16 = 16%$$

- ▶ apare interferența între diversele unde reflectate
- ▶ se adună campurile nu puterile
- ▶ lamele antireflexive



Transmisia puterii intre medii

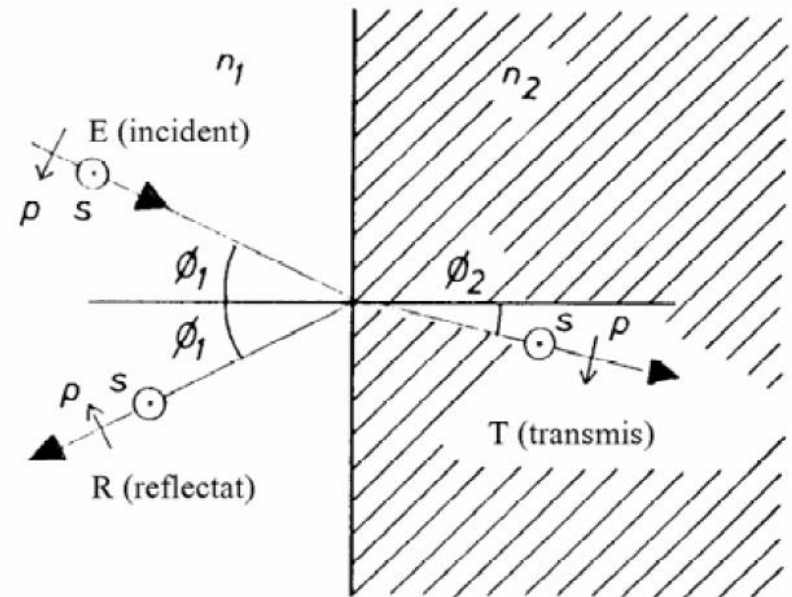
- ▶ incidenta oblica
- ▶ reflexiile in amplitudine a campului:

$$r_s = -\frac{\sin(\phi_1 - \phi_2)}{\sin(\phi_1 + \phi_2)}$$

$$r_p = \frac{\tan(\phi_1 - \phi_2)}{\tan(\phi_1 + \phi_2)}$$

$$t_s = \frac{2 \sin \phi_2 \cos \phi_1}{\sin(\phi_1 + \phi_2)}$$

$$t_p = \frac{2 \sin \phi_2 \cos \phi_1}{\sin(\phi_1 + \phi_2) \cos(\phi_1 - \phi_2)}$$



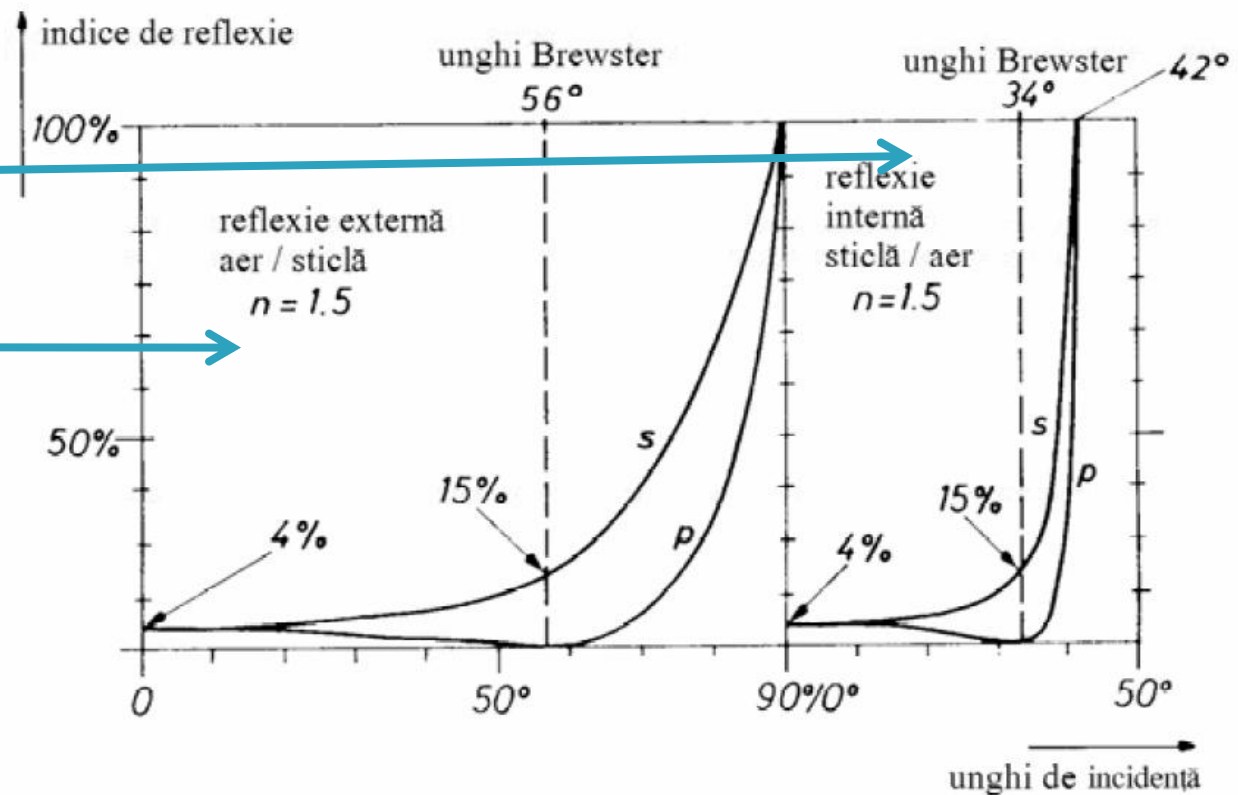
Unghi Brewster

- ▶ transmisia totala a polarizarii p
- ▶ lumina reflectata este total polarizata (s)

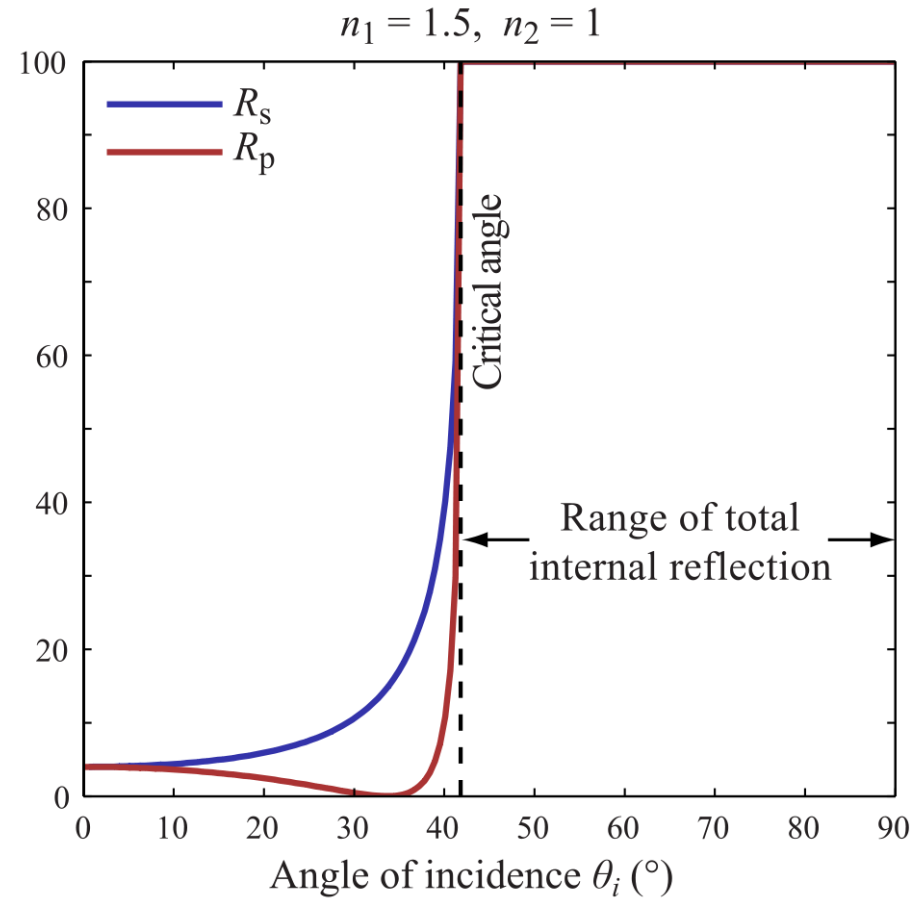
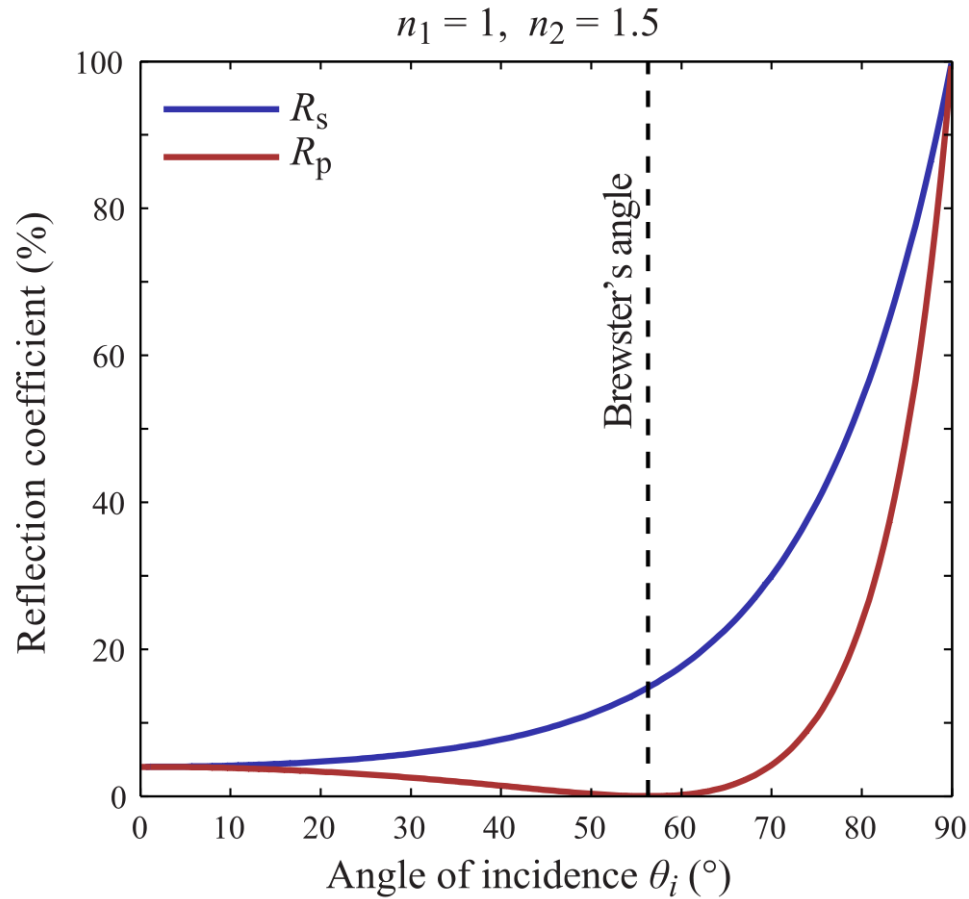
$$\phi_B = \arctan\left(\frac{n_2}{n_1}\right)$$

$$\phi_B = 34^\circ$$

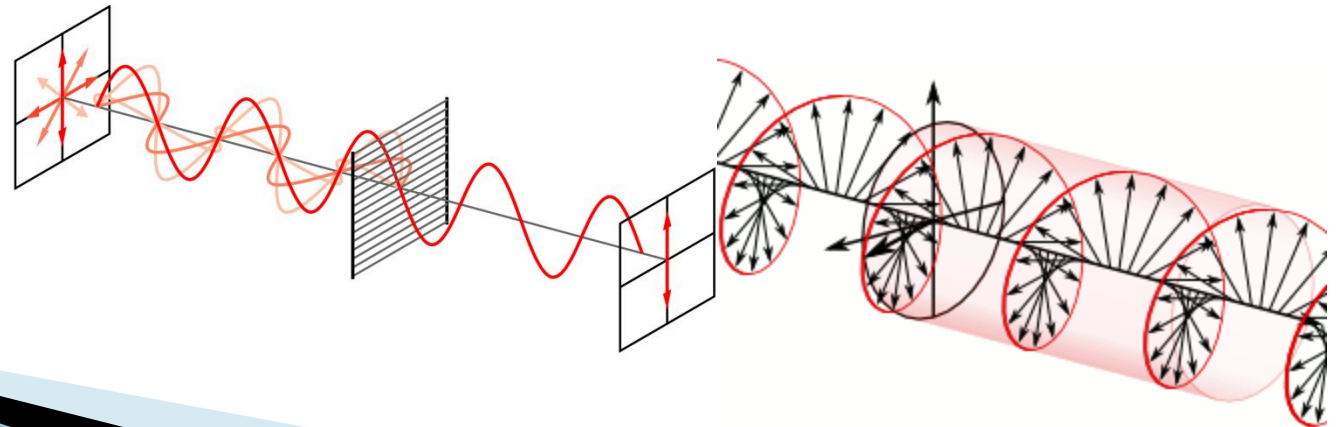
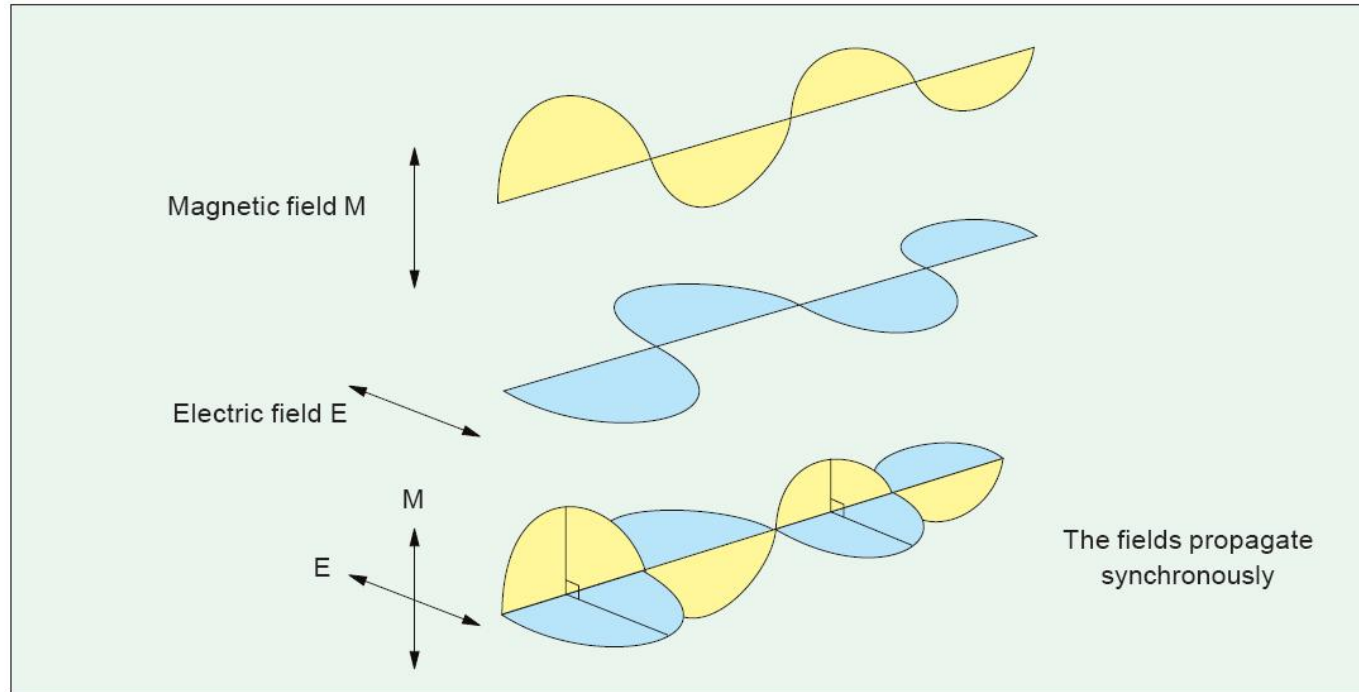
$$\phi_B = 56^\circ$$



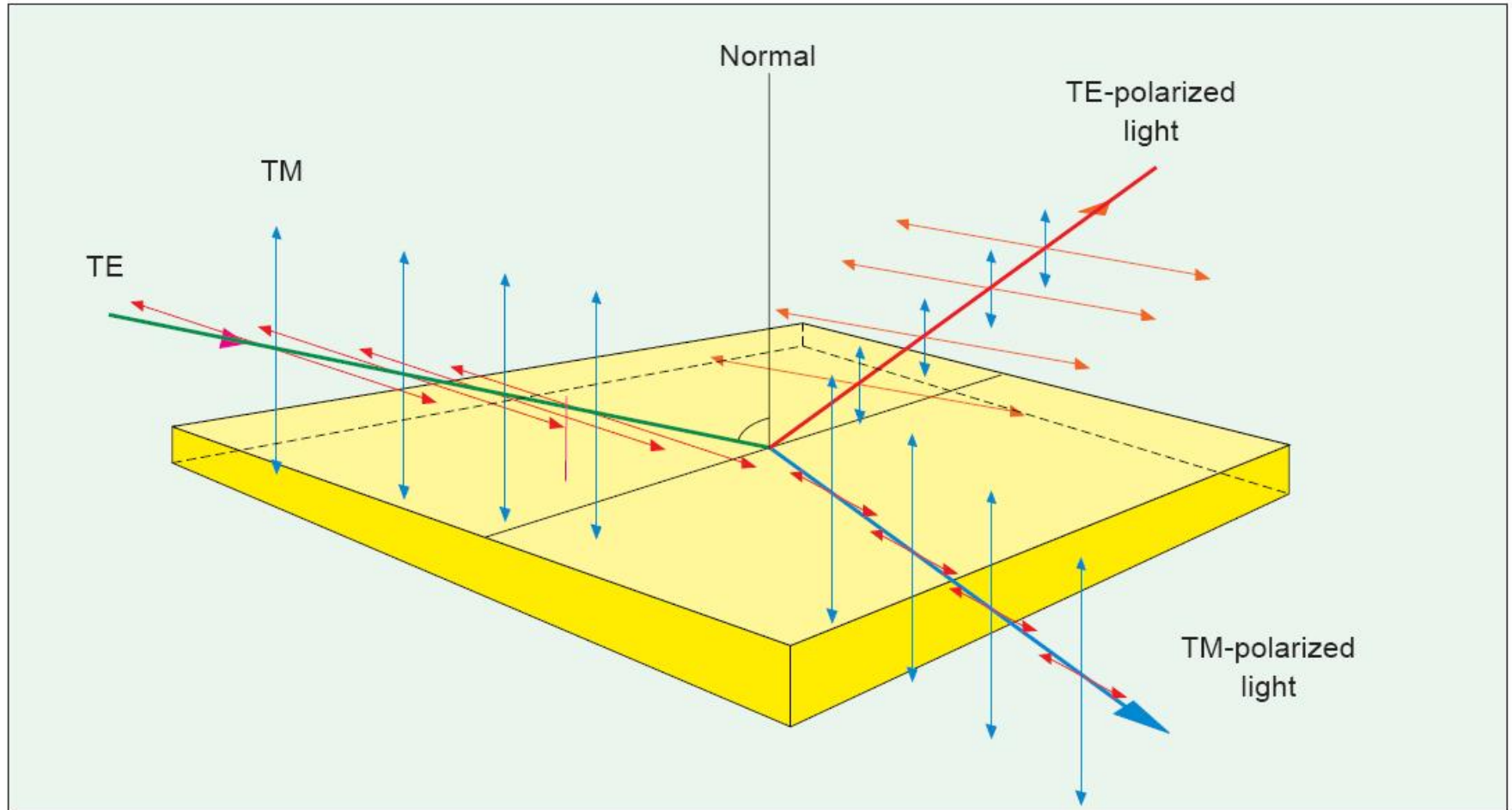
Unghi Brewster



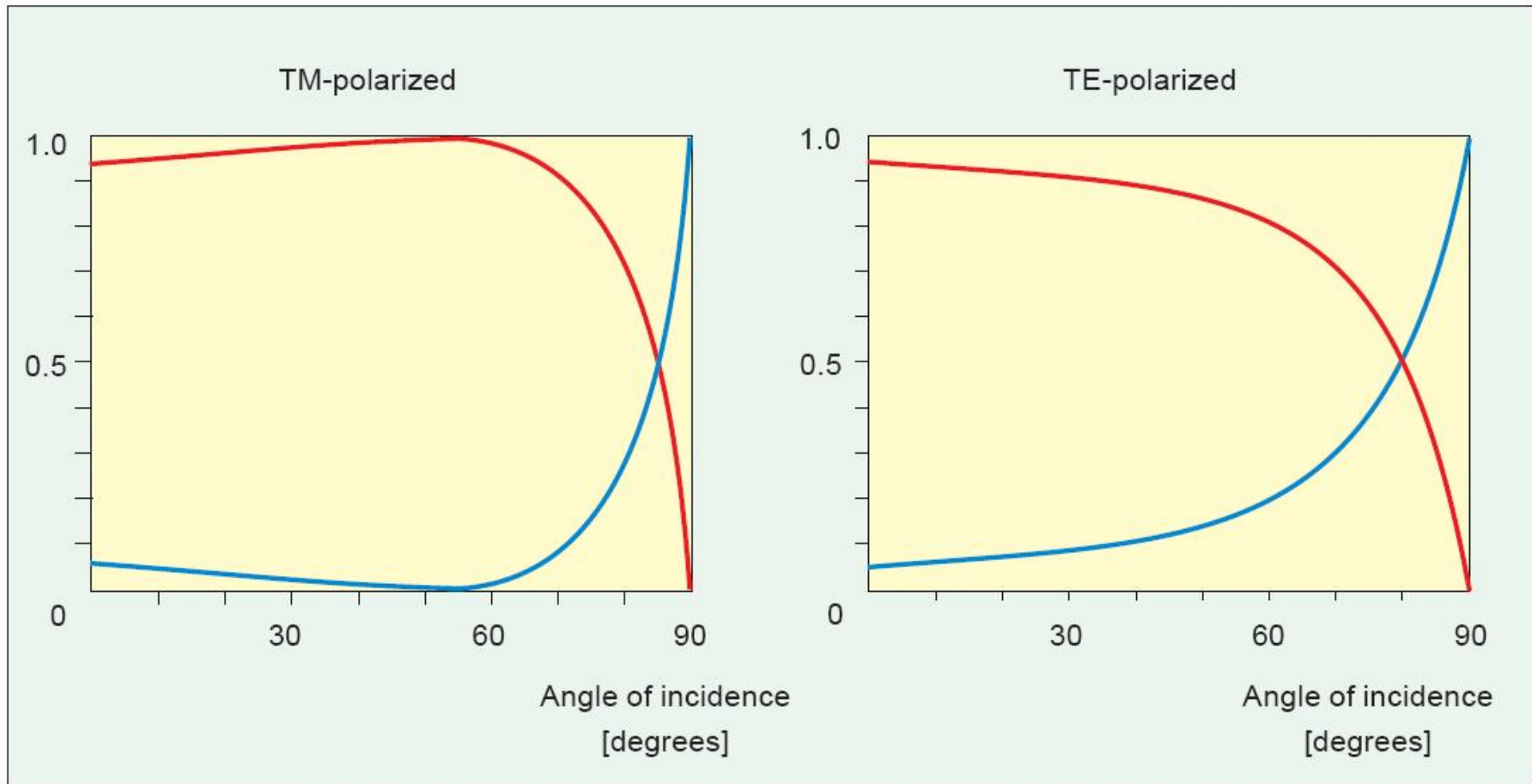
Polarizarea luminii



Polarizarea luminii



Polarizarea luminii



Contact

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