Microstrip Antennas
Cap.4

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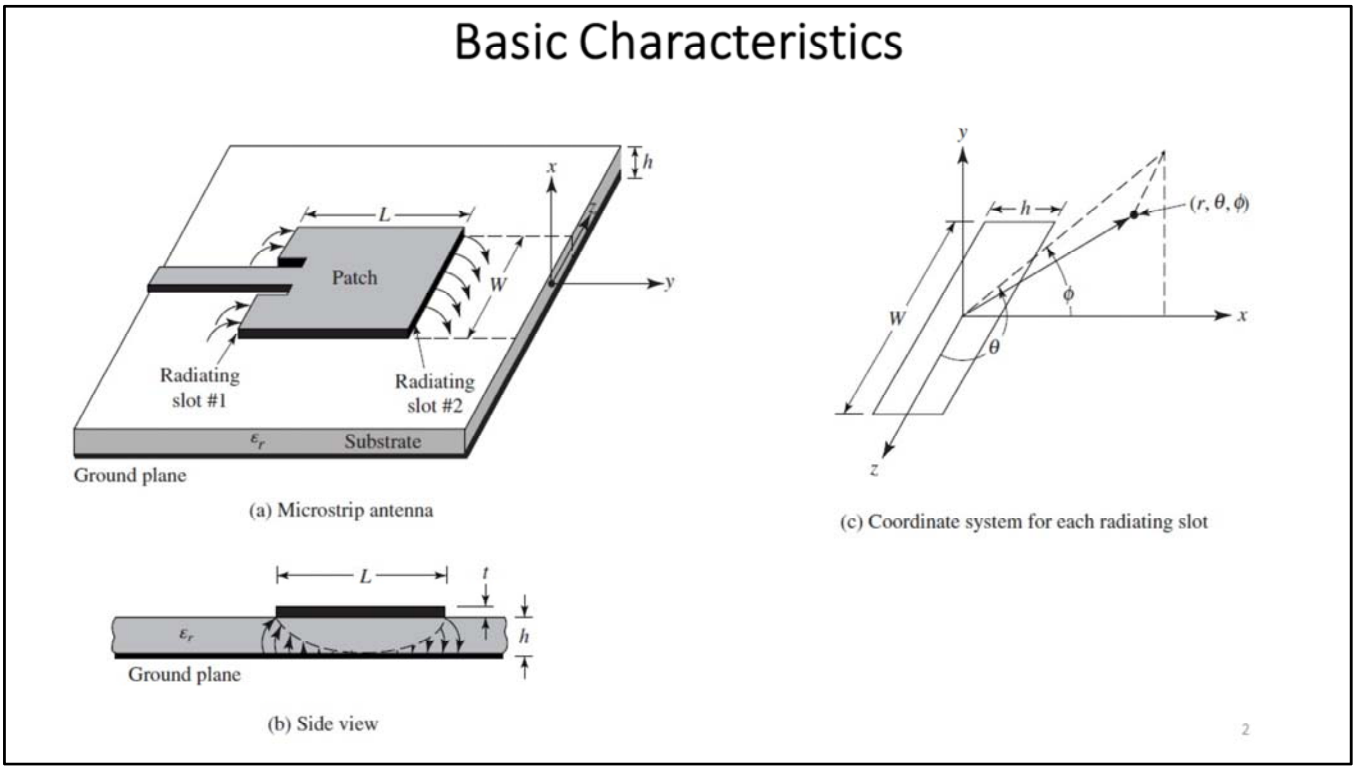


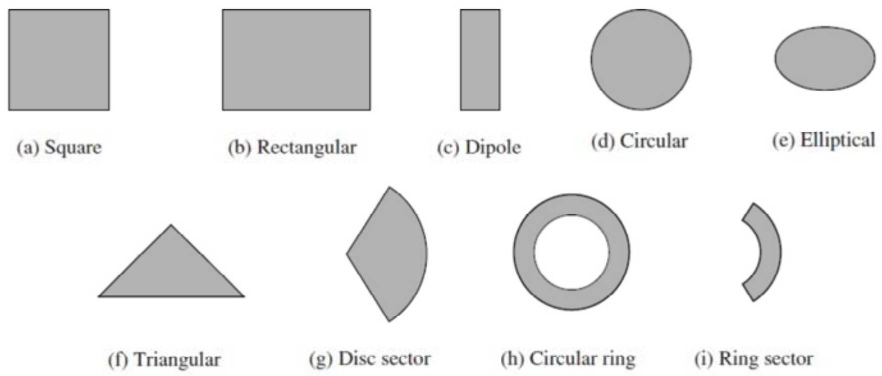
(a) Microstrip antenna
(b) Side view
(c) Coordinate system for each radiating slot

Microstrip antennas, as shown in Figure (a), consist of a very thin ($t \ll \lambda_0$, where λ_0 is the free-space wavelength) metallic strip (patch) placed a small fraction of a wavelength ($h \ll \lambda_0$, usually $0.003\lambda_0 \le h \le 0.05\lambda_0$) above a ground plane. The microstrip patch is designed so its pattern maximum is normal to the patch (broadside radiator). This is accomplished by properly choosing the mode (field configuration) of excitation beneath the patch. End-fire radiation can also be accomplished by judicious mode selection.

For a rectangular patch, the length L of the element is usually $\lambda_0/3 < L < \lambda_0/2$. The strip (patch) and the ground plane are separated by a dielectric sheet (referred to as the substrate), as shown in Figure (a).

There are numerous substrates that can be used for the design of microstrip antennas, and their dielectric constants are usually in the range of $2.2 \le \varepsilon_r \le 12$. The ones that are most desirable for good antenna performance are thick substrates whose dielectric constant is in the lower end of the range because they provide better efficiency, larger bandwidth, loosely bound fields for radiation into space, but at the expense of larger element size. Thin substrates with higher dielectric constants are desirable for microwave circuitry because they require tightly bound fields to minimize undesired radiation and coupling, and lead to smaller element sizes; however, because of their greater losses, they are less efficient and have relatively smaller bandwidths. Since microstrip antennas are often integrated with other microwave circuitry, a compromise has to be reached between good antenna performance and circuit design.

Basic Characteristics – cont.



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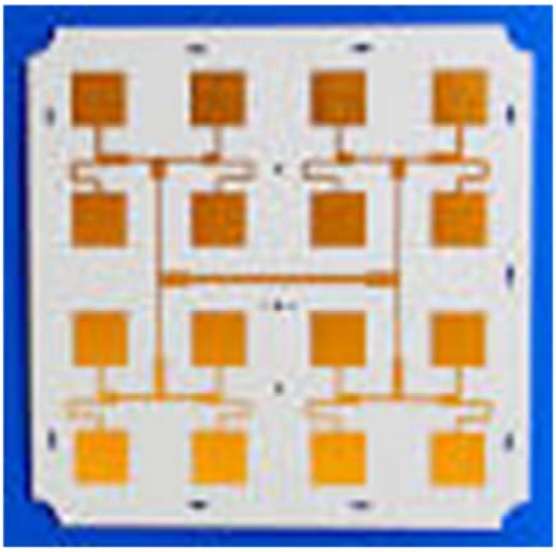
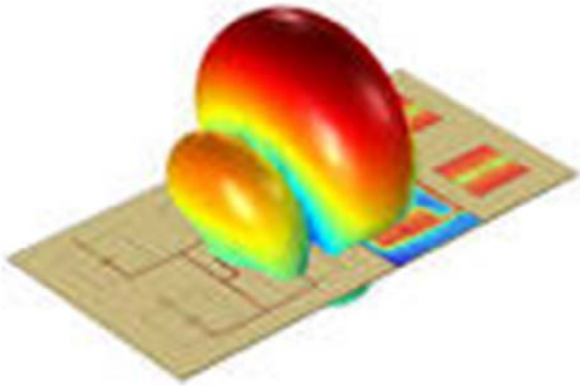
Often microstrip antennas are also referred to as patch antennas. The radiating elements and the feed lines are usually photoetched on the dielectric substrate. The radiating patch may be square, rectangular, thin strip (dipole), circular, elliptical, triangular, or any other configuration. These and others are illustrated in Figure. Square, rectangular, dipole (strip), and circular are the most common because of ease of analysis and fabrication, and their attractive radiation characteristics, especially low cross-polarization radiation.

Basic Characteristics – cont.





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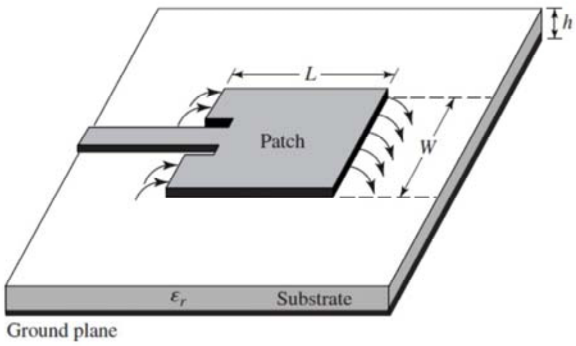
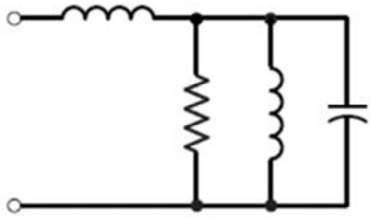
Microstrip dipoles are attractive because they inherently possess a large bandwidth and occupy less space, which makes them attractive for arrays. Linear and circular polarizations can be achieved with either single elements or arrays of microstrip antennas. Arrays of microstrip elements, with single or multiple feeds, may also be used to introduce scanning capabilities and achieve greater directivities.

Feeding Methods



(a) Microstrip line feed

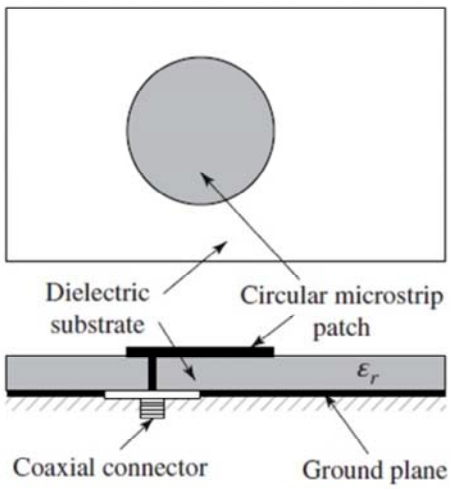


(a) Microstrip line

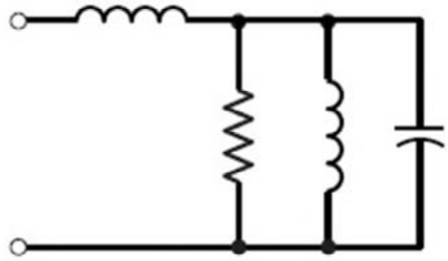
5

There are many configurations that can be used to feed microstrip antennas. The microstrip feed line is also a conducting strip, usually of much smaller width compared to the patch. The microstrip-line feed is easy to fabricate, simple to match by controlling the inset position and rather simple to model. However as the substrate thickness increases, surface waves and spurious feed radiation increase, which for practical designs limit the bandwidth (typically 2–5%).

Feeding Methods – cont.



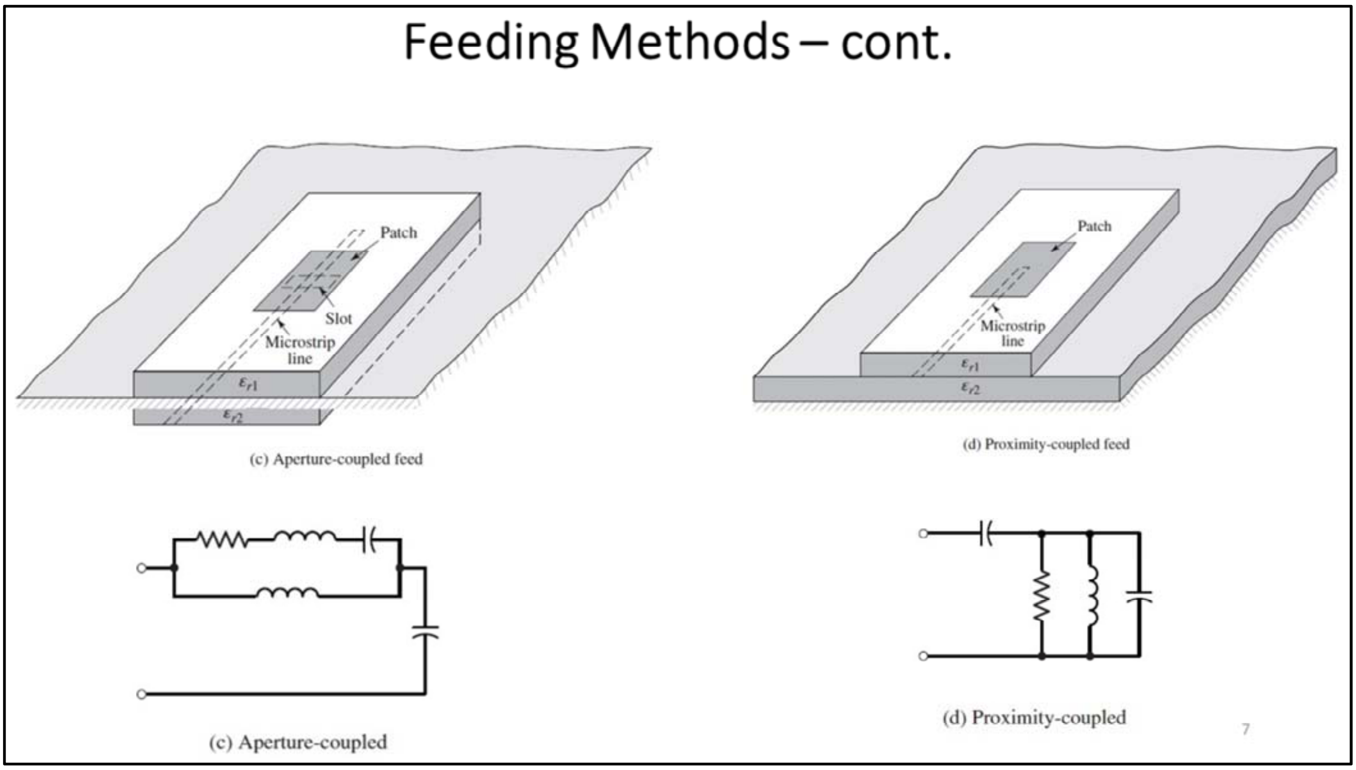
(b) Probe feed



(b) Probe

6

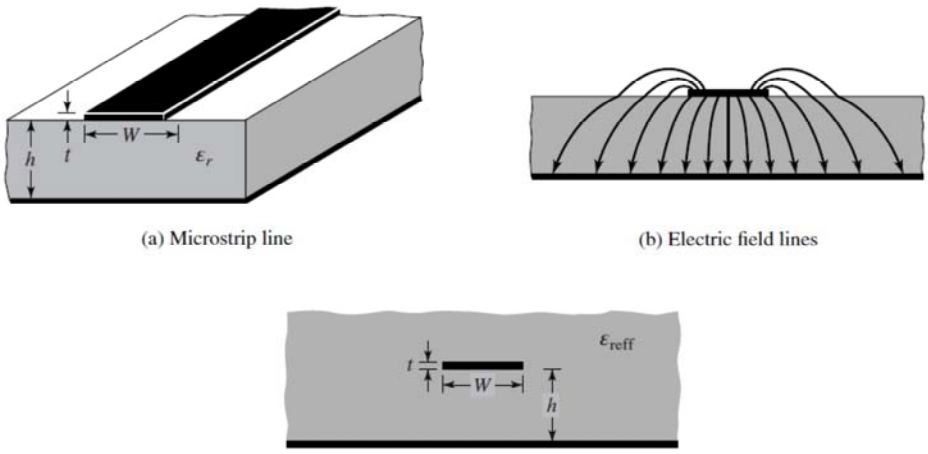
Coaxial-line feeds, where the inner conductor of the coax is attached to the radiation patch while the outer conductor is connected to the ground plane, are also widely used. The coaxial probe feed is also easy to fabricate and match, and it has low spurious radiation. However, it also has narrow bandwidth and it is more difficult to model, especially for thick substrates ($h > 0.02\lambda_0$).
Both the microstrip feed line and the probe possess inherent asymmetries which generate higher order modes which produce cross-polarized radiation. To overcome some of these problems, noncontacting aperture-coupling feeds, as shown have been introduced.



The aperture coupling of Figure (c) is the most difficult of all four to fabricate and it also has narrow bandwidth. However, it is somewhat easier to model and has moderate spurious radiation. The aperture coupling consists of two substrates separated by a ground plane. On the bottom side of the lower substrate there is a microstrip feed line whose energy is coupled to the patch through a slot on the ground plane separating the two substrates. This arrangement allows independent optimization of the feed mechanism and the radiating element. Typically a high dielectric material is used for the bottom substrate, and thick low dielectric constant material for the top substrate. The ground plane between the substrates also isolates the feed from the radiating element and minimizes interference of spurious radiation for pattern formation and polarization purity. For this design, the substrate electrical parameters, feed line width, and slot size and position can be used to optimize the design. Typically matching is performed by controlling the width of the feed line and the length of the slot. The coupling through the slot can be modeled using the theory of Bethe.

Of the four feeds described here, the proximity coupling has the largest bandwidth (as high as 13%), is somewhat easy to model and has low spurious radiation. However its fabrication is somewhat more difficult. The length of the feeding stub and the width-to-line ratio of the patch can be used to control the match.

RECTANGULAR PATCH



(a) Microstrip line

(b) Electric field lines

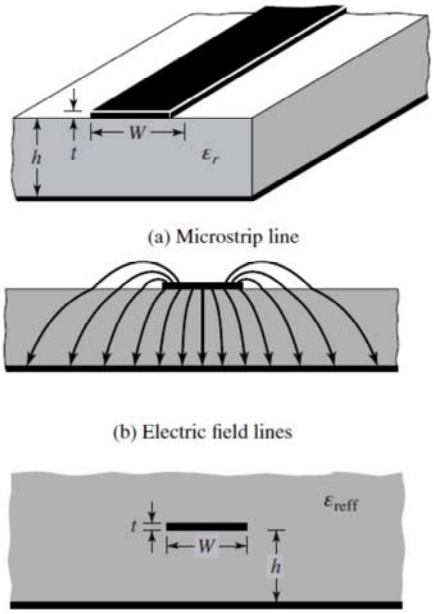
(c) Effective dielectric constant

8

A rectangular microstrip antenna can be represented as an array of two *radiating* narrow apertures (slots), each of width *W* and height *h*, separated by a distance *L*. Basically the transmission-line model represents the microstrip antenna by two slots, separated by a low-impedance *Zc* transmission line of length *L*. Because the dimensions of the patch are finite along the length and width, the fields at the edges of the patch undergo fringing. This is illustrated along the length in Figures (a,b) for the two radiating slots of the microstrip antenna. The same applies along the width. The amount of fringing is a function of the dimensions of the patch and the height of the substrate. For the principal *E*-plane (*xy*-plane) fringing is a function of the ratio of the length of the patch *L* to the height *h* of the substrate (*L/h*) and the dielectric constant ε_r of the substrate. Since for microstrip antennas $L/h \gg 1$, fringing is reduced; however, it must be taken into account because it influences the resonant frequency of the antenna. The same applies for the width.

For a microstrip line shown in Figure (a), typical electric field lines are shown in Figure (b). This is a nonhomogeneous line of two dielectrics; typically the substrate and air. As can be seen, most of the electric field lines reside in the substrate and parts of some lines exist in air. As $W/h \gg 1$ and $\varepsilon_r \gg 1$, the electric field lines concentrate mostly in the substrate. Fringing in this case makes the microstrip line look wider electrically compared to its physical dimensions. Since some of the waves travel in the substrate and some in air, an *effective dielectric constant* ε_{reff} is introduced to account for fringing and the wave propagation in the line.

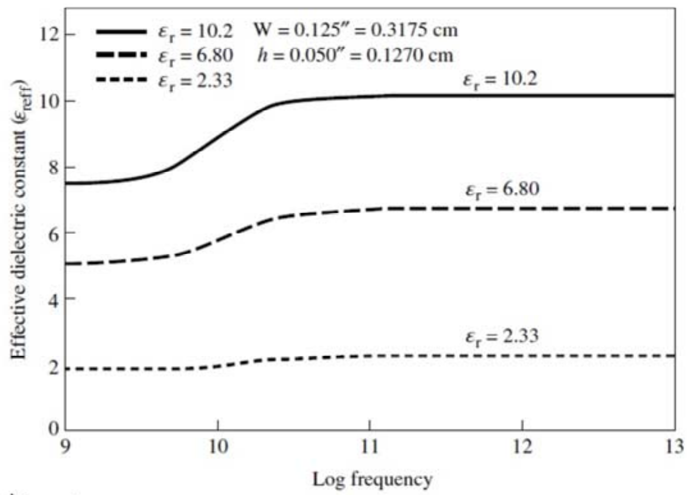
The effective dielectric constant



(a) Microstrip line

(b) Electric field lines

(c) Effective dielectric constant



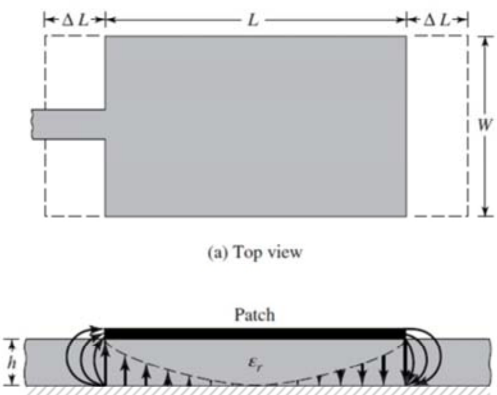
$W/h > 1$

$$\varepsilon_{reff} = \frac{\varepsilon_r + 1}{2} + \frac{\varepsilon_r - 1}{2}\left[1 + 12\frac{h}{W}\right]^{-1/2} \qquad (1)$$

9

To introduce the effective dielectric constant, let us assume that the center conductor of the microstrip line with its original dimensions and height above the ground plane is embedded into one dielectric, as shown in Figure (c). The effective dielectric constant is defined *as the dielectric constant of the uniform dielectric material so that the line of Figure (c) has identical electrical characteristics, particularly propagation constant, as the actual line of Figure (a)*. For a line with air above the substrate, the effective dielectric constant has values in the range of $1 < \varepsilon_{reff} < \varepsilon_r$. For most applications where the dielectric constant of the substrate is much greater than unity ($\varepsilon_r \gg 1$), the value of ε_{reff} will be closer to the value of the actual dielectric constant ε_r of the substrate.
The effective dielectric constant is also a function of frequency. As the frequency of operation increases, most of the electric field lines concentrate in the substrate. Therefore the microstrip line behaves more like a homogeneous line of one dielectric (only the substrate), and the effective dielectric constant approaches the value of the dielectric constant of the substrate. Typical variations, as a function of frequency, of the effective dielectric constant for a microstrip line with three different substrates are shown in right Figure.
For low frequencies the effective dielectric constant is essentially constant. At intermediate frequencies its values begin to monotonically increase and eventually approach the values of the dielectric constant of the substrate. The initial values (at low frequencies) of the effective dielectric constant are referred to as the <u>*static values*</u>, and they are given by Eq.(1)

Effective Length, Resonant Frequency, and Effective Width



(a) Top view

(b) Side view

$$\frac{\Delta L}{h} = 0.412 \frac{\left(\varepsilon_{reff} + 0.3\right)\left(\frac{W}{h} + 0.264\right)}{\left(\varepsilon_{reff} - 0.258\right)\left(\frac{W}{h} + 0.8\right)} \quad (2)$$

$$L_{eff} = L + 2\Delta L \quad (3)$$

$$\left(f_r\right)_{010} = \frac{1}{2L\sqrt{\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = \frac{c}{2L\sqrt{\varepsilon_r}} \quad (4)$$

$$q = \frac{\left(f_{rc}\right)_{010}}{\left(f_r\right)_{010}} \quad (5a)$$

$$\left(f_{rc}\right)_{010} = \frac{1}{2L_{eff}\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} = \frac{1}{2\left(L + 2\Delta L\right)\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} =$$

$$= q\frac{1}{2L\sqrt{\varepsilon_r}\sqrt{\mu_0\varepsilon_0}} = q\frac{c}{2L\sqrt{\varepsilon_r}} \quad (5)$$

10

Because of the fringing effects, electrically the patch of the microstrip antenna looks greater than its physical dimensions. For the principal *E*-plane (*xy*-plane), this is demonstrated in Figure, where the dimensions of the patch along its length have been extended on each end by a distance Δ*L*, which is a function of the effective dielectric constant $\varepsilon_{\text{reff}}$ and the width-to-height ratio (*W/h*). A very popular and practical approximate relation for the normalized extension of the length is Eq.(2).
Since the length of the patch has been extended by Δ*L* on each side, the effective length of the patch is now Eq.(3)($L = \lambda/2$ for dominant TM_{010} mode with no fringing).
For the dominant TM_{010} mode, the resonant frequency of the microstrip antenna is a function of its length. Usually it is given by Eq.(4), where c is the speed of light in free space. Since Eq.(4) does not account for fringing, it must be modified to include edge effects and should be computed using Eq.(5), where q is given by Eq.(5a).
The *q* factor is referred to as the *fringe factor* (length reduction factor). As the substrate height increases, fringing also increases and leads to larger separations between the radiating edges and lower resonant frequencies. The designed resonant frequency, based on fringing, is lower as the patch looks longer, as indicated in Figure. The resonant frequency decrease due to fringing is usually 2–6%.

Design

$$W = \frac{1}{2f_r\sqrt{\mu_0\varepsilon_0}}\sqrt{\frac{2}{\varepsilon_r+1}} = \frac{c}{2f_r}\sqrt{\frac{2}{\varepsilon_r+1}} \quad (6)$$

$$L = \frac{1}{2f_r\sqrt{\varepsilon_{reff}}\sqrt{\mu_0\varepsilon_0}} - 2\Delta L \quad (7)$$

$$L \approx (0.47\cdots0.49)\frac{\lambda_0}{\sqrt{\varepsilon_r}} = (0.47\cdots0.49)\lambda_d \quad (7a)$$

11

Based on the simplified formulation that has been described, a design procedure is outlined which leads to practical designs of rectangular microstrip antennas. The procedure assumes that the specified information includes the dielectric constant of the substrate (ϵ_r), the resonant frequency (f_r), and the height of the substrate *h*. The procedure is as follows:

Specify : ε_r, f_r (in Hz), and *h*

Determine : *W*, *L*

Design procedure:

1. For an efficient radiator, a practical width that leads to good radiation efficiencies is Eq.(6)

2. Determine the effective dielectric constant of the microstrip antenna using Eq.(1).

3. Once *W* is found using Eq.(6), determine the extension of the length Δ*L* using Eq.(2).

4. The actual length of the patch can now be determined by solving Eq.(5) for *L*, or Eq.(7). Typical lengths of microstrip patches vary as in Eq.(7a), where λ_d is the wavelength in the dielectric.

The smaller the dielectric constant of the substrate, the larger is the fringing; thus, the length of the microstrip patch is smaller. In contrast, the larger the dielectric constant, the more tightly the fields are held within the substrate; thus, the fringing is smaller and the length is longer and closer to half-wavelength in the dielectric.

Example 1

Design a rectangular microstrip antenna using a substrate (RT/duroid 5880) with dielectric constant of 2.2, h=0.1588 cm (0.0625 inches) so as to resonate at 10 GHz.

Solutie : Using eq.(6), the width W of the patch is

$$W = \frac{3 \cdot 10^8}{2(10)10^9}\sqrt{\frac{2}{2.2+1}} = 1.186\, cm\,(0.467\, in)$$

The effective dielectric constant of the patch is found using Eqq.(1), or

$$\varepsilon_{reff} = \frac{2.2+1}{2} + \frac{2.2-1}{2}\left(1+12\frac{0.1588}{1.186}\right)^{-1/2} = 1.972$$

The extended incremental length of the patch ΔL is, using Eq.(2)

$$\Delta L = 0.1588(0.412)\frac{(1.972+0.3)\left(\frac{1.186}{0.1588}+0.264\right)}{(1.972-0.258)\left(\frac{1.186}{0.1588}+0.8\right)} = 0.081 cm\,(0.032 in)$$

12

Example 1 – cont.

The actual length L of the patch is found using Eq.(3), or

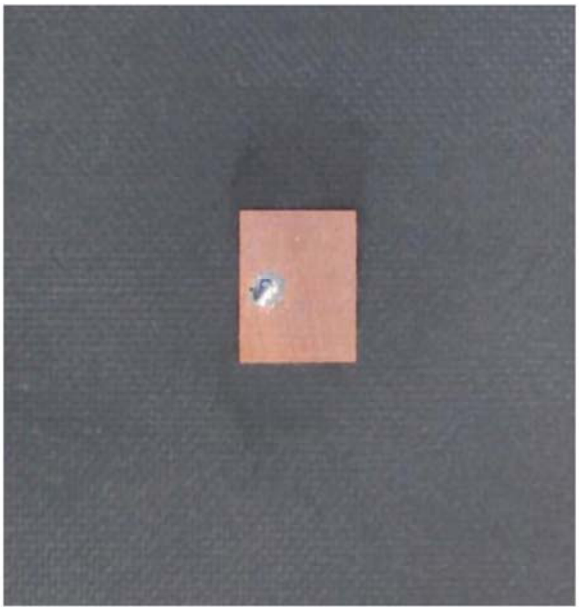
$$L = \frac{\lambda}{2} - 2\Delta L = \frac{3}{2(100)\sqrt{1.972}} - 2(0.081) = 0.906cm(0.357in)$$

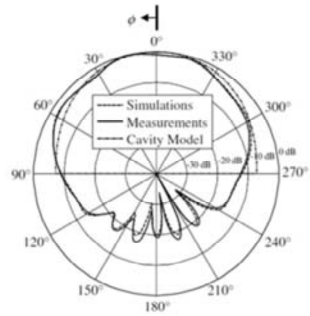
Finally the effective length is

$$L_e = L + 2\Delta L = \frac{\lambda}{2} = 1.068cm(0.421in)$$

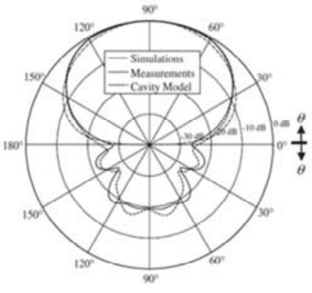
An experimental rectangular patch based on this design was built and tested. It is probe fed from underneath by coaxial line and is shown in Figure (a). Its principal E- and H- plane patterns are displayed in Figure (c,d) for $f_0 = 9.8GHz$.

Example 1 – cont.



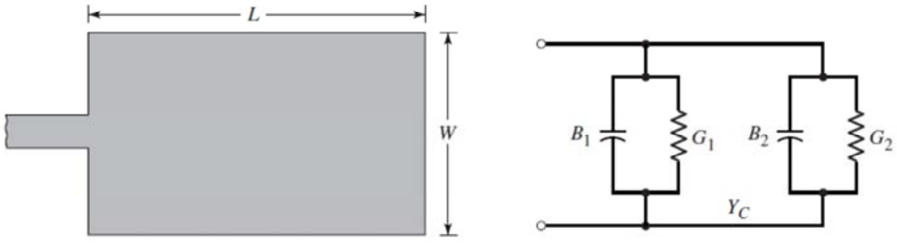


(c) E-plane ($\theta = 90°$)



(d) H-plane ($\phi = 0°$)

Conductance



(a) Rectangular patch (b) Transmission model equivalent

$$Y_1 = G_1 + jB_1 \quad (8)$$

$$G_1 = \frac{W}{120\lambda_0}\left[1 - \frac{1}{24}(k_0 h)^2\right] \quad \frac{h}{\lambda_0} < \frac{1}{10} \quad (8a)$$

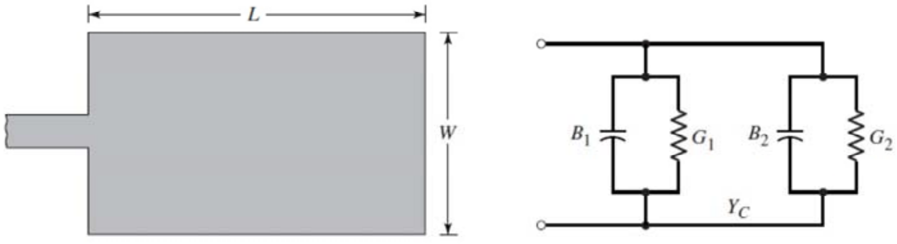
$$B_1 = \frac{W}{120\lambda_0}\left[1 - 0.636\, ln(k_0 h)\right] \quad \frac{h}{\lambda_0} < \frac{1}{10} \quad (8b)$$

$$Y_2 = Y_1, \quad G_2 = G_1, \quad B_2 = B_1 \quad (9)$$

15

Each radiating slot is represented by a parallel equivalent admittance *Y* (with conductance *G* and susceptance *B*). This is shown in Figure. The slots are labeled as #1 and #2. The equivalent admittance of slot #1, based on an infinitely wide, uniform slot, is derived in bibliography and it is given by Eq.(8), where for a slot of finite width *W we have Eq.(8a,b).*
Since slot #2 is identical to slot #1, its equivalent admittance is Eq.(9).

Conductance – cont.



(a) Rectangular patch (b) Transmission model equivalent

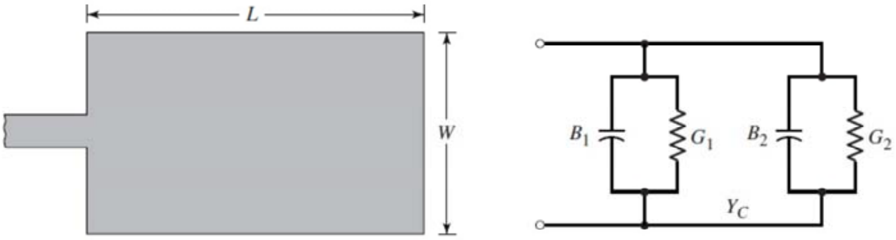
$$G_1 = \frac{2P_{rad}}{|V_0|^2} \quad (10)$$

$$P_{rad} = \frac{|V_0|^2}{2\pi\eta_0} \int_0^{\pi} \left[\frac{sin\left(\frac{k_0 W}{2} cos\,\theta\right)}{cos\,\theta} \right]^2 sin^3\theta d\theta \quad (11)$$

16

The conductance of a single slot can also be obtained by using the field expression derived by the cavity model (see bibliography for details). In general, the conductance is defined as Eq.(10). Using the electric field, the radiated power is written as Eq.(11).

Conductance – cont.



(a) Rectangular patch (b) Transmission model equivalent

$$G_1 = \frac{I_1}{120\pi^2} \quad (12)$$

$$I_1 = \int_0^{\pi} \left[\frac{\sin\left(\frac{k_0 W}{2}\cos\theta\right)}{\cos\theta} \right]^2 \sin^3\theta d\theta = -2 + \cos(X) + X S_i(X) + \frac{\sin(X)}{X} \quad (12a)$$

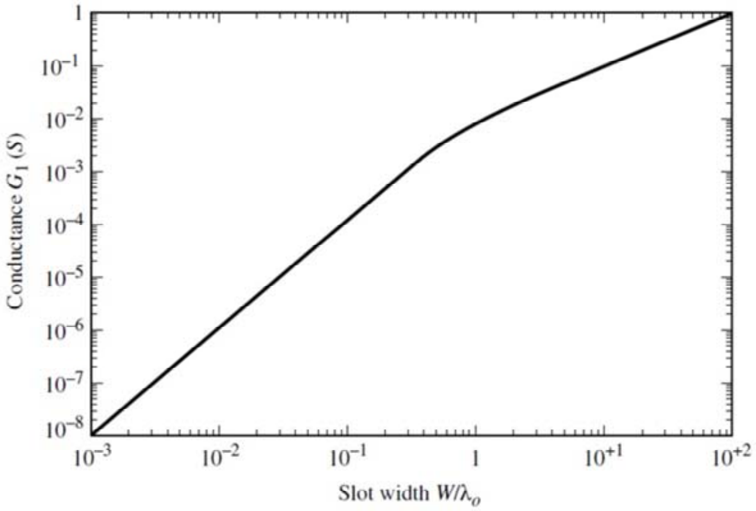
$$X = k_0 W \quad (12b)$$

$$S_i(X) \overset{def}{=} \int_0^X \frac{\sin(t)}{t} dt$$

17

Therefore the conductance of Eq.(10) can be expressed as Eq.(12), where we have Eq.(12a,b).

Conductance – cont.



$$G_1 = \begin{cases} \dfrac{1}{90}\left(\dfrac{W}{\lambda_0}\right)^2 & W \ll \lambda_0 \\ \dfrac{1}{120}\left(\dfrac{W}{\lambda_0}\right) & W \gg \lambda_0 \end{cases} \qquad (13)$$

18

Asymptotic values of Eqs.(12) and (12a) are given in Eq.(13).
The values of Eq.(13) for $W \gg \lambda_0$ are identical to those given by Eq.(8a) for $h \ll \lambda_0$. A plot of G as a function of W/λ_0 is shown in Figure.

Resonant Input Resistance

$$\tilde{Y}_2 = \tilde{G}_2 + j\tilde{B}_2 = G_1 - jB_1 \quad (14)$$

$$\tilde{G}_2 = G_1 \quad (14a)$$

$$\tilde{B}_2 = -B_1 \quad (14b)$$

$$Y_{in} = Y_1 + \tilde{Y}_2 = 2G_1 \quad (15)$$

$$Z_{in} = \frac{1}{Y_{in}} = R_{in} = \frac{1}{2G_1} \quad (16)$$

$$R_{in} = \frac{1}{2(G_1 \pm G_{12})} \quad (17)$$

$$G_{12} = \frac{1}{|V_0|^2} \Re e \iint_S \boldsymbol{E}_1 \times \boldsymbol{H}_2^* \cdot d\boldsymbol{s} \quad (18)$$

$$G_{12} = \frac{1}{120\pi^2} \int_0^{\pi} \left[\frac{sin\left(\frac{k_0 W}{2} cos\,\theta\right)}{cos\,\theta} \right]^2 J_0(k_0 L\, sin\,\theta) sin^3\theta\, d\theta \quad (18a)$$

19

The total admittance at slot #1 (input admittance) is obtained by transferring the admittance of slot#2 from the output terminals to input terminals using the admittance transformation equation of transmission lines. Ideally the two slots should be separated by $\lambda/2$ where λ is the wavelength in the dielectric (substrate). However, because of fringing the length of the patch is electrically longer than the actual length. Therefore the actual separation of the two slots is slightly less than $\lambda/2$. If the reduction of the length is properly chosen using Eq.(2) (typically $0.47\lambda < L < 0.49\lambda$), the transformed admittance of slot #2 becomes Eq.(14) or Eq.(14a,b). Therefore the total resonant input admittance is real and is given by Eq.(15). Since the total input admittance is real, the resonant input impedance is also real, or Eq.(16).

The resonant input resistance, as given by Eq.(16), does not take into account mutual effects between the slots. This can be accomplished by modifying Eq.(16) to Eq.(17) where the plus (+) sign is used for modes with odd (antisymmetric) resonant voltage distribution beneath the patch and between the slots while the minus (−) sign is used for modes with even (symmetric) resonant voltage distribution. The mutual conductance is defined, in terms of the far-zone fields, as Eq.(18), where $\mathbf{E}_1$ is the electric field radiated by slot #1, $\mathbf{H}_2$ is the magnetic field radiated by slot #2, V_0 is the voltage across the slot, and the integration is performed over a sphere of large radius. It can be shown that G_{12} can be calculated using Eq.(18a), where J_0 is the Bessel function of the first kind of order zero. For typical microstrip antennas, the mutual conductance obtained using Eq.(18a) is small compared to the self conductance G_1 of Eq.(8a) or Eq.(12).

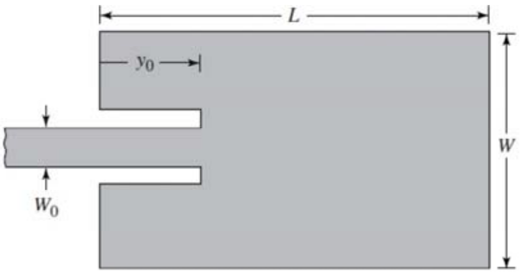
Resonant Input Resistance – cont.

$$R_{in} = 90\frac{(\varepsilon_r)^2}{\varepsilon_r - 1}\left(\frac{L}{W}\right) \quad (18b)$$

20

An alternate approximate expression for the input impedance, R_{in}, for a resonant patch, is Eq.(18b).
This expression is valid for thin substrates ($h \ll \lambda_0$). Equation (17), and Eq.(18b) give reasonable results for the input resistance *Rin*, although they are not identical.
As shown by Eq.(8a) and Eq.(17), the input resistance is not strongly dependent upon the substrate height *h*. In fact for very small values of *h*, such that $k_0 h \ll 1$, the input resistance is not dependent on *h*.
It is apparent from Eq.(8a) and Eq.(17) that the resonant input resistance can be decreased by increasing the width *W* of the patch. This is acceptable as long as the ratio of *W / L* does not exceed 2 because the aperture efficiency of a single patch begins to drop, as *W / L* increases beyond 2.

Matching Techniques



(a) Recessed microstrip-line feed

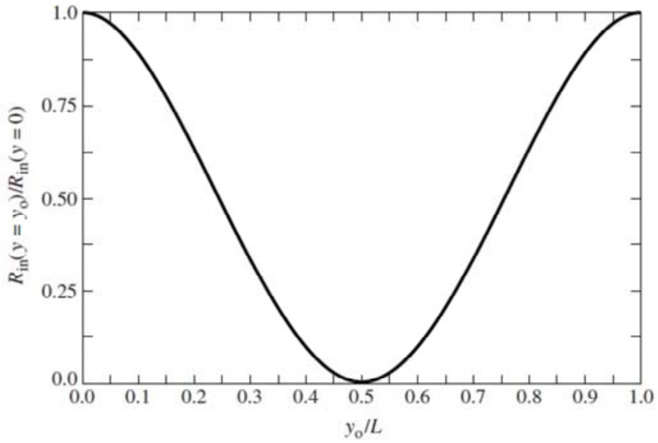
$$Z_c = \begin{cases} \dfrac{60}{\sqrt{\varepsilon_{reff}}} ln\left[\dfrac{8h}{W_0} + \dfrac{W_0}{4h}\right], & \dfrac{W_0}{h} \le 1 \quad (19a) \\ \dfrac{120\pi}{\sqrt{\varepsilon_{reff}}\left[\dfrac{W_0}{h} + 1.393 + 0.667\, ln\left(\dfrac{W_0}{h} + 1.444\right)\right]}, & \dfrac{W_0}{h} > 1 \quad (19b) \end{cases}$$

$$R_{in(y=y_0)} = \frac{1}{2(G_1 \pm G_{12})}\left[cos^2\left(\frac{\pi}{L}y_0\right) + \frac{G_1^2 + B_1^2}{Y_c^2} sin^2\left(\frac{\pi}{L}y_0\right) - \frac{B_1}{Y_c} sin\left(\frac{2\pi}{L}y_0\right)\right] \quad (20)$$

21

The resonant input resistance, as calculated by Eq.(17), is referenced at slot #1. However, it has been shown that the resonant input resistance can be changed by using an inset feed, recessed a distance y_0 from slot #1, as shown in Figure (a). This technique can be used effectively to match the patch antenna using a microstrip-line feed whose characteristic impedance is given by Eq.(19), where W_0 is the width of the microstrip line, as shown in Figure. Using modal-expansion analysis, the input resistance for the inset feed is given approximately by Eq.(20), where $Yc = 1 / Zc$.

Matching Techniques – cont.



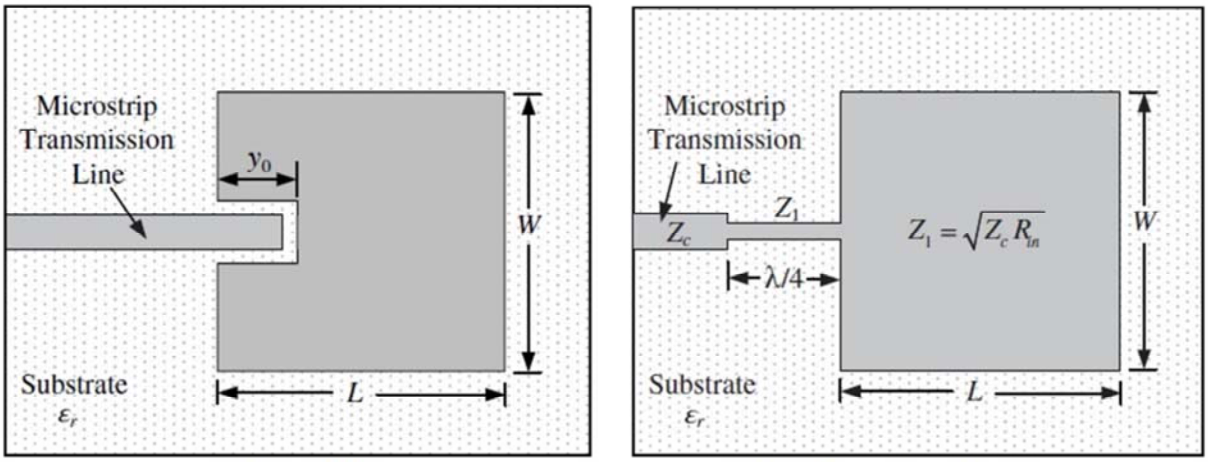
(b) Normalized input resistance

$$R_{in(y=y_0)} = \frac{1}{2(G_1 \pm G_{12})} cos^2\left(\frac{\pi}{L} y_0\right) = R_{in}(y=0) cos^2\left(\frac{\pi}{L} y_0\right) \quad (20a)$$

22

Since for most typical microstrips $G_1 / Yc \ll 1$ and $B_1 / Yc \ll 1$, Eq.(20) reduces to Eq.(20a). A plot of the normalized value of Eq.(20a) is shown in Figure (b). It is apparent from Eq.(20a) and Figure (b) that the maximum value occurs at the edge of the slot (y_0 = 0) where the voltage is maximum and the current is minimum; typical values are in the 150–300 ohms. The minimum value (zero) occurs at the center of the patch ($y_0 = L / 2$) where the voltage is zero and the current is maximum. As the inset feed point moves from the edge toward the center of the patch the resonant input impedance decreases monotonically and reaches zero at the center. When the value of the inset feed point approaches the center of the patch ($y_0 = L / 2$), the cos2($\pi y_0 / L$) function varies very rapidly; therefore the input resistance also changes rapidly with the position of the feed point. To maintain very accurate values, a close tolerance must be preserved.

Matching Techniques – cont.



(a) Coupled (b) λ/4 impedance transformer

23

Other matching techniques, aside from the recessed microstrip of previous Figure, are the *coupled recessed microstrip* and *the λ ⁄ 4 impedance transformer* of Figure (a,b). The *Rin* in Figure (b) is the input resistance at the leading edge of the resonant patch; it must be real.

Example 2

A microstrip antenna with overall dimensions of L = 0.906 cm (0.357 inches) and W = 1.186 cm (0.467 inches), substrate with height h = 0.1588 cm (0.0625 inches) and dielectric constant of $\varepsilon_r = 2.2$, is operating at 10 GHz. Find:

a. The input impedance

b. The position of the inset feed point where the input impedance is 50Ω.

Solution :

$$\lambda_0 = \frac{30}{10} = 3cm$$

Using Eq.(12) and (12a)

$G_1 = 0.00157$ siemens

which compares with $G_1 = 0.00328$ using Eq.(8a). Using Eq.(18a)

$G_{12} = 6.1683 \times 10^{-4}$

Example 2 – cont.

Using Eq.(17) with the (+) sign because of the odd distribution between the radiating slots for the dominant TM_{010} mode

$R_{in} = 228.3508\Omega$

Since the input impedance at the leading raddiating edge of the patch is 228.3508 ohms while the desired impedance is 50 ohms, the inset feed point distance y_0 is obtained using Eq.(20a). Thus

$$50 = 228.3508\cos^2\left(\frac{\pi}{L}y_0\right)$$

or

$$y_0 = 0.3126 \text{ cm } (0.123 \text{ inches})$$

25

Directivity

$$D_0 = \frac{U_{max}}{U_0} = \frac{4\pi U_{max}}{P_{rad}} \quad (21) \qquad U_{max} = \frac{|V_0|^2}{2\eta_0\pi^2}\left(\frac{\pi W}{\lambda_0}\right)^2 \quad (22) \qquad P_{rad} = \frac{|V_0|^2}{2\eta_0\pi}\int_0^{\pi}\left[\frac{sin\left(\frac{k_0 W}{2}cos\theta\right)}{cos\theta}\right]^2 sin^3 d\theta \quad (23)$$

$$D_0 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{1}{I_1} \quad (24)$$

$$I_1 = \int_0^{\pi}\left[\frac{sin\left(\frac{k_0 W}{2}cos\theta\right)}{cos\theta}\right]^2 sin^3 d\theta = \left[-2 + cos(X) + XS_i(X) + \frac{sin(X)}{X}\right] \quad (25a)$$

$$X = k_0 W \quad (25b)$$

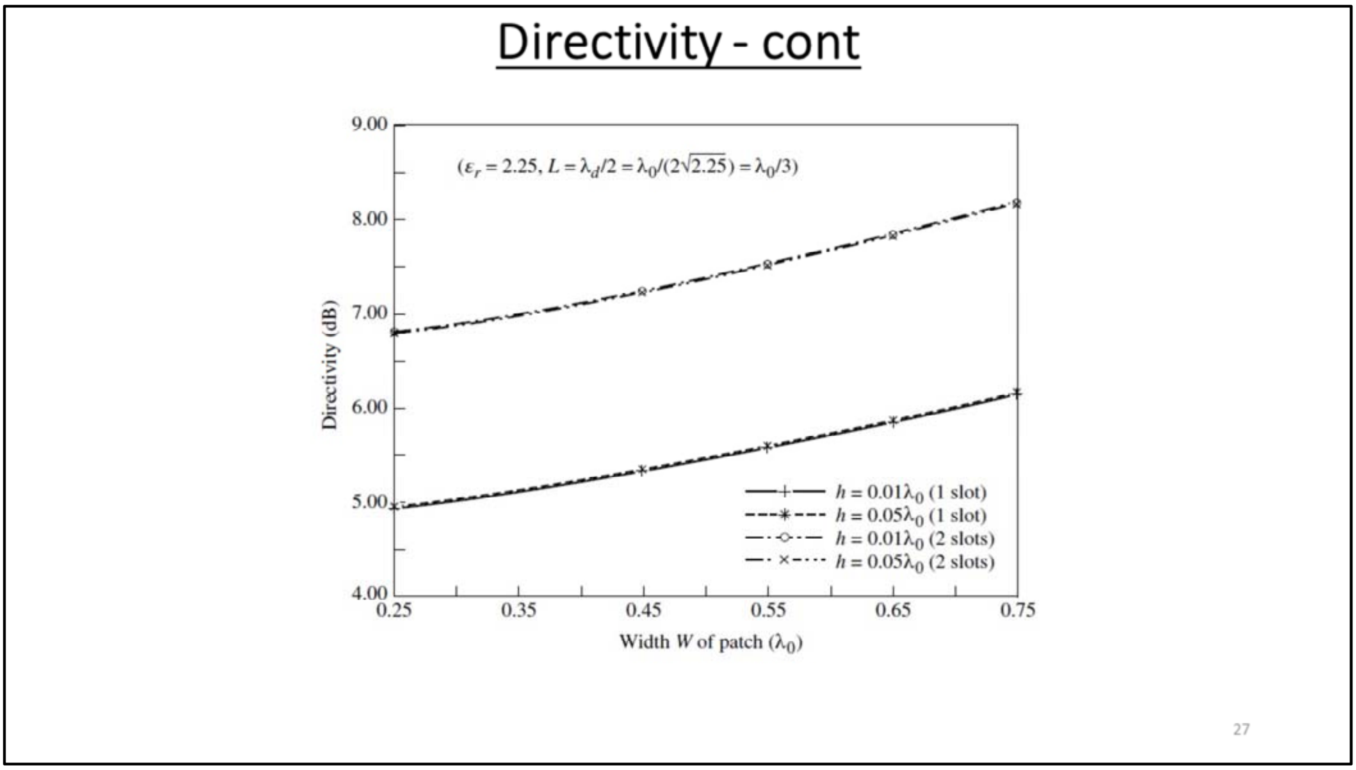
$$D_0 = \begin{cases} 3.3(dimensionless) = 5.2dB, W \ll \lambda_0 & (26a) \\ 4\left(\frac{W}{\lambda_0}\right), W \gg \lambda_0 & (26b) \end{cases}$$

26

As for every other antenna, the directivity is one of the most important figures-of-merit whose definition is given in Eq.(21).

Single Slot ($k_0h \ll 1$)

Using the electric field, the maximum radiation intensity and radiated power can be written, respectively, as Eq.(22) and (23). Therefore, the directivity of a single slot can be expressed as Eq.(24), where Eqs.(25). Asymptotically the values of Eq.(25) vary as Eq.(26).



Single Slot ($k_0h \ll 1$)

The directivity of a single slot can be computed using the equations (24) and (25a). Plots of the directivity of a single slot for h = $0.01\lambda_0$ and $0.05\lambda_0$ as a function of the width of the slot are shown in Figure. It is evident that the directivity of a single slot is not influenced strongly by the height of the substrate, as long as it is maintained electrically small.

Directivity - cont

$$D_2 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{\pi}{I_2} = \frac{2}{15G_{rad}}\left(\frac{W}{\lambda_0}\right)^2 \quad (27)$$

$$I_2 = \int_0^{\pi}\int_0^{\pi}\left[\frac{sin\left(\frac{k_0 W}{2}cos\theta\right)}{cos\theta}\right]^2 sin^3\, cos^2\left(\frac{k_0 L_e}{2}sin\theta\, sin\phi\right) d\theta d\phi \quad (28)$$

$$D_2 = D_0 D_{AF} = D_0 \frac{2}{1+g_{12}} \quad (29)$$

$$D_{AF} = \frac{2}{1+g_{12}} \overset{g_{12}\ll 1}{=} 2 \quad (29a)$$

$$D_2 = \begin{cases} 6.6(\text{dimensionless}) = 8.2dB, & W \ll \lambda_0 \\ 8\left(\frac{W}{\lambda_0}\right), & W \gg \lambda_0 \end{cases} \quad (30)$$

28

Two Slots ($k_0h \ll 1$)

For two slots the directivity can be written as Eq.(27), where G_{rad} is the radiation conductance and I_2 is given by Eq.(28). The total broadside directivity D_2 for the two radiating slots, separated by the dominant TM_{010}^{x} mode field (antisymmetric voltage distribution), can also be written as Eq.(29), where:

D_0 = directivity of single slot [as given by Eq.(24) and (25a)]

D_{AF} = directivity of array factor AF [$AF = \cos\left(\frac{k_0 L_e}{2}\sin\theta\sin\phi\right)$]

g_{12} = normalized mutual conductance = G_{12} / G_1

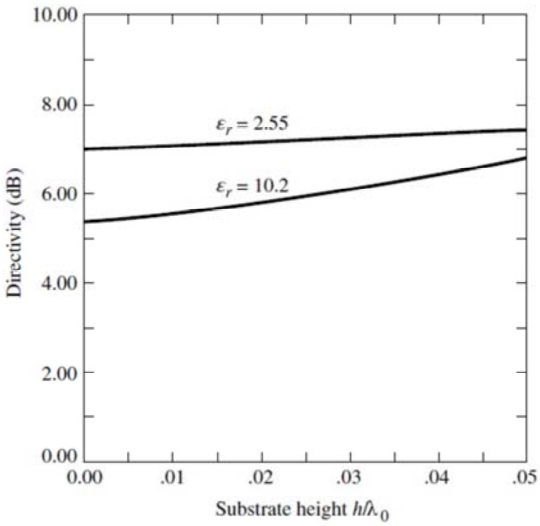
The normalized mutual conductance g_{12} can be obtained using Eqs.(12)+(12a), and (18a).

Computed values based on Eq.(18a) show that usually $g_{12} \ll 1$; thus Eq.(29a) is usually a good approximation to Eq.(29).

Asymptotically, the directivity of two slots (microstrip antenna) can be expressed as Eq.(30).

The directivity of the microstrip antenna can now be computed using Eqs.(27) and (28).

Directivity - cont



29

Plots of directivity of a microstrip antenna, modeled by two slots, for $h = 0.01\lambda_0$ and $0.05\lambda_0$ are shown plotted as a function of the width of the patch (W / λ_0) in Figure/slide 26. It is evident that the directivity is not a strong function of the height, as long as the height is maintained electrically small. About 2 dB difference is indicated between the directivity of one and two slots. A typical plot of the directivity of a patch for a fixed resonant frequency as a function of the substrate height (h / λ_0), for two different dielectrics, is shown in Figure.

Example 3

For the rectangular microstrip antenna of Examples 1 and 2, with overall dimensions of L = 0.906 cm and W = 1.186 cm, substrate height h = 0.1588 cm, and dielectric constant of ε_r=2.2, center frequency of 10 GHz, find the directivity based on Eq.(29) and (29a). Compare with the values obtained using Eq.(27) and (28).

Solution :

From the solution of Example 2

$G_1 = 0.00157$ siemens

$G_{12} = 6.1683 \times 10^{-4}$ siemens

$g_{12} = G_{12}/G_1 = 0.3921$

Using Eq.(29a)

$$D_{AF} = \frac{2}{1+g_{12}} = \frac{2}{1+0.3921} = 1.4367 = 1.5736dB$$

30

Example 3 – cont.

Using (24) and (25a)

$I_1 = 1.863$

$$D_0 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{1}{I_1} = 3.312 = 5.201 dB$$

According to Eq.(29)

$D_2 = D_0 D_{AF} = 3.312(1.4367) = 4.7584 = 6.7746 dB$

Using Eq.(27)

$I_2 = 3.59801$

Finnaly, using Eq.(27)

$$\mathrm{D}_2 = \left(\frac{2\pi W}{\lambda_0}\right)^2 \frac{\pi}{I_2} = 5.3873 (dim\ ensionless) = 7.314 dB$$

There is about 0.5 dB difference between the directivity computed using Eq.(27) and Eq.(29); the onne based on Eq.(27) is more accurate. 31